WESTERN CANADA LINEAR ALGEBRA MEETING (WCLAM)

Program - Abstracts - Participants

University of Calgary

May 25-26, 2024

Organizing Committee

Shaun Fallat, Hadi Kharaghani, Steve Kirkland, Sarah Plosker,
Michael Tsatsomeros, Pauline van den Driessche
Honorary Organizer: Peter Lancaster

Local Organizers:
Shaun Fallat, Matthew Greenberg

Funding

The WCLAM Organizing Committee gratefully acknowledges the generous support for this meeting provided by: *The Pacific Institute for the Mathematical Sciences* and the *Department of Mathematics & Statistics at the University of Calgary*.

Invited Speakers

Amir Amiraslani, Capilano University
Panayiotis J. Psarrakos, National Technical University of Athens
Francoise Tissuer, University of Manchester

Location

Mathematical Sciences (MS) Building (Rooms 431, 461)
University of Calgary

1 Meeting Program (Math. Sciences Bldg. # 431)

Saturday, May 25, 2024

08:00-08:45 **Registration**

08:45-09:00 **Welcome & Information**

Chair: Michael Tsatsomeros

09:00-09:45 **Panayiotis Psarrakos (Invited)**, *From matrices to matrix polynomials*

09:50-10:10 **Ion Zaballa (virtual)**, *Phase synchronization and filters for quadratic systems*

10:10-10:30 **D. Steven Mackey (virtual)**, *On the sign characteristic of higher degree Hermitian matrix polynomials*

10:30-10:50 **Fernando de Terán Vergara (virtual)**, *Regular vs singular matrix polynomials*

10:50-11:10 **Break** *(MS 461)*

Chair: Shaun Fallat

11:10-11:35 **Tin Yau Tam (virtual)**, *Yamamoto-Nayak's theorem and its extension to Lie groups*

11:40-12:05 **Jason Moliteirno (virtual)**, *Entries of the bottleneck matrices for maximal outerplanar graphs*

12:05-13:30 **Lunch (on your own)**

Chair: Pauline van den Driessche

13:30-13:55 **Hermie Monterde**, *Quantum walks on graph operations*

14:00-14:25 **Benjamin Clark**, *The nonnegative inverse eigenvalue problem is solvable and the algorithm to solve it exists. So why is the problem unsolved?*

14:30-14:55 **Jared Brannan**, *Algebraic curves that annihilate combinatorial arrays*

15:00-15:20 **Break** *(MS 461)*

Chair: Steve Kirkland

15:20-16:05 **Amir Amiraslani (Invited)**, *Normalization method over tropical semirings*

16:10-16:35 **Garrett Kepler**, *Independent sets and graph energy*

16:40-17:05 **Michael Overton**, *Crouzeix's conjecture*

18:00 **Informal Banquet–Notable** *133-4611 Bowness Road NW*

Sunday, May 26, 2024

Chair: Shaun Fallat

09:00-09:45	**Francoise Tisseur (Invited & virtual)**, *Deflation strategies for nonlinear eigenvalue problems*
9:50-10:15	**Sarah Plosker (virtual)**, *Birkhoff–James orthogonality in the trace norm*
10:20-10:45	**Andre Ran (virtual)**, *Unbounded Toeplitz operators: invertibility and Riccati equations*
10:45-11:05	**Group photo and break** *(MS 461)*

Chair: Hadi Kharaghani

11:05-11:30	**Piyush Verma**, *Spectral properties of token graphsAn introduction to strong properties*
11:35-12:00	**Avleen Kaur**, *Estimating the minimum positive eigenvalue of PSD matrices*
12:05-12:30	**Colin Garnett**, *An introduction to strong properties*
12:35-1:00	**Ludovick Bouthat**, *The submultiplicativity of matrix norms induced by random vectors*
1:00 –	**Closing remarks & Tributes (Fallat and others...)**

Other Relevant Conference Information:

- University Map: Available on the conference website.
- Parking is available, please visit the University of Calgary website for more information.
- Dining: Please visit the University of Calgary website for more information.
- Group Photo: Will take place on Sunday during the morning break.
- Zoom Link: Provided via email from S. Fallat after registration, within 48 hours of the meeting commencing.

2 Abstracts for talks (alphabetical by speaker)

Normalization Method over Tropical Semirings
Amir Amiraslani

Solving linear systems of equations over "tropical semirings" finds its applications in problems related to automata theory, optimization theory, manufacturing systems, telecommunication networks, parallel processing systems, and traffic control. The main purpose of this talk is to introduce an algorithmic approach for examining the behaviour of linear systems over tropical semirings and solving them if possible. We use a method called "normalization" to construct an associated normalized matrix, which gives a technique for solving the system. In fact, determining the "column minimum elements" of the associated normalized matrix provides useful and thorough information about the existence of solutions as well as the solutions themselves of any linear system, $AX = b$, and every system equivalent to it in the given tropical semiring. As such, we are able to investigate the solvability of a wide range of linear systems. Importantly, the column minimum elements lead us to obtain a "maximal solution" of the system. If solutions exist, the method can also determine the number of degrees of freedom of the system. Moreover, using the results, we present a procedure to determine the column rank and the row rank of a matrix.

The talk is mainly based on this paper which is a joint work with some of my colleagues.

Algebraic Curves that Annihilate Combinatorial Arrays
Jared Brannan

Riordan arrays are matrices indexed by pairs of natural numbers that have been used to identify novel combinatorial identities. In this talk, Riordan arrays are extended to be indexed by pairs of integers. Next, the recurrences a given array satisfies are identified with linear operators which map the array to an array of all zeros. I then show that these operators form a ring isomorphic to a polynomial ring. Finally, I use the algebraic varieties associated with these ideals to determine structural properties of recurrences for certain classes of arrays.

The Submultiplicativity of Matrix Norms Induced by Random Vectors
Ludovick Bouthat

In a recent article, Chavez, Garcia and Hurley introduced a new family of norms $\|\cdot\|_{\mathbf{X},d}$ on the space of $n \times n$ complex matrices which are induced by random vectors $\mathbf{X}$ having finite d-moments. In this talk, the interesting properties of these norms are exhibited, and recent progress concerning the submultiplicativity of these norms is presented. In particular, we shall see that they are submultiplictive, as long as the entries of $\mathbf{X}$ have finite p-moments for $p = \max\{2 + \varepsilon, d\}$.

The Nonnegative Inverse Eigenvalue Problem is Solvable and the Algorithm to Solve it exists. So Why is the Problem Unsolved?

Benjamin Clark

The nonnegative inverse eigenvalue problem (NIEP) asks what the necessary and sufficient conditions are such that a list of complex numbers forms the spectra of a nonnegative matrix. In this talk, I will give some background into the NIEP and its related subproblems. Next, I will discuss how the NIEP forms a semialgebraic set and why it can then be solved by polynomial inequalities. Finally, I will give an overview of some algorithms that can give a solution to the NIEP and outline why we can't directly use them.

An Introduction to Strong Properties

Colin Garnett

Strong Properties for sign patterns have been of interest in studying their inertia. The Strong Multiplicity Property and Strong Spectral Property were introduced in a paper by Shaun Fallat, Tracy Hall, Jephian Lin, and Bryan Shader. These properties have been used to answer questions about the spectrum of graphs. I will discuss the general techniques used to construct strong properties as well as some analogous nonsymmetric strong properties.

Estimating the Minimum Positive Eigenvalue of PSD Matrices

Avleen Kaur

An extensive body of literature addresses the estimation of eigenvalues of the sum of two symmetric matrices, $P + Q$, in relation to the eigenvalues of P and Q. Recently, we introduced two novel lower bounds on the minimum eigenvalue, $\lambda_{\min}(P+Q)$, under the conditions that matrices P and Q are symmetric positive semi-definite (PSD) and their sum $P + Q$ is non-singular. These bounds rely on the Friedrichs angle between the range spaces of matrices P and Q, which are denoted by $\mathcal{R}(P)$ and $\mathcal{R}(Q)$, respectively. In addition, both results led to the derivation of several new lower bounds on the minimum singular value of full-rank matrices. We extend these insights to estimate the minimum positive eigenvalue of $P + Q$, $\lambda_{\min}(P + Q)$, even if $P + Q$ is singular, in terms of the minimum positive eigenvalues of P and Q, namely $\lambda_{\min}(P)$ and $\lambda_{\min}(Q)$. Our approach leverages angles between specific subspaces of $\mathcal{R}(P)$ and $\mathcal{R}(Q)$, meticulously chosen to yield a positive lower bound. Additionally, we illustrate these concepts through relevant examples.

Independent Sets and Graph Energy

Garrett Kepler

The sum of the moduli of the adjacency spectra of a graph, the adjacency energy, has been shown to connect linear algebra and the electron energy of molecules. This talk will cover a conjectured lower bound relating the energy of a graph and the size of its largest independent set. A solution for graphs with proportionally large independent sets will be shown and the consequences explored.

On the Sign Characteristic of Higher Degree Hermitian Matrix Polynomials

D. Steven Mackey

The sign characteristic is a structural feature of Hermitian matrix polynomials, invariant under unimodular congruence, that is important for both theory and applications. It consists of a plus or minus sign attached to each elementary divisor associated with a real or infinite eigenvalue. When all eigenvalues are simple, these signs can be naturally ordered to form a *sign sequence*. For an $n \times n$ Hermitian polynomial of degree d with dn simple real eigenvalues, there are 2^{dn} conceivable sign sequences, many of which cannot be realized by any degree d Hermitian polynomial. This talk explores the possibility of characterizing exactly which sign sequences are realizable, and which are not.

Entries of the Bottleneck Matrices for Maximal Outerplanar Graphs

Jason J. Molitierno

In this talk, we consider the entries of the bottleneck matrices of maximal outerplanar graphs. We recall patterns involving the entries of bottleneck matrices for trees and find similar but more general patters for maximal outerplanar graphs. We first establish patterns involving the diagonal entries of bottleneck matrices for maximal outerplanar graphs. We then investigate patterns involving the off-diagonal entries. We consider patterns of entries increasing and decreasing and prove results involving concavity. We close with examples illustrating our results.

Quantum Walks on Graph Operations

Hermie Monterde

A continuous quantum walk on a graph X is determined by the unitary matrix $U(t) = \exp(itA)$, where A is the adjacency matrix of X. We say that *perfect state transfer* (PST) occurs between two vertices u and v of X at time τif $|U(\tau)_{u,v}| = 1$, and we say that vertex u of X is *sedentary* if for some constant C such that $0 < C \leq 1$, we have $|U(\tau)_{u,u}| > C$ for all t. In this talk, we survey old and new results about PST and vertex sedentariness under Cartesian and direct products, joins and blow-up graphs. This is joint work with Hiranmoy Pal (NIT Rourkela), Stephen Kirkland (University of Manitoba) and Sarah Plosker (Brandon University).

Crouzeix's Conjecture

Micheal Overton

Crouzeix's conjecture is among the most intriguing developments in matrix theory in recent years. Made in 2004 by Michel Crouzeix, it postulates that, for any polynomial p and any matrix A, $\|p(A)\| \leq 2 \max\{|p(z)| : z \in W(A)\}$, where the norm is the 2-norm and $W(A)$ is the field of values (numerical range) of A, that is the set of points attained by v^*Av for some vector v of unit length. Crouzeix proved in 2007 that the inequality above holds if 2 is replaced by 11.08, and in 2016 this was greatly improved by Palencia, replacing 2 by $1 + \sqrt{2}$. Furthermore, it is known that the conjecture holds in a number of special cases. We use nonsmooth optimization to investigate the conjecture numerically by locally minimizing the "Crouzeix ratio", defined as the quotient with numerator the right-hand side and denominator the left-hand side of the conjectured inequality. We use Chebfun to compute the boundary of the fields of values and BFGS for the optimization, computing the Crouzeix ratio at billions of data points. We also present local nonsmooth variational analysis of the Crouzeix ratio at conjectured global minimizers. All our results strongly support the truth of Crouzeix's conjecture.

Birkhoff–James Orthogonality in the Trace Norm

Sarah Plosker

Birkhoff–James orthogonality was introduced in 1947 to provide a definition of orthogonality in normed vector spaces that do not have an inner product. We characterize when a given complex Hermitian matrix is Birkhoff–James orthogonal in the trace norm to a given (Hermitian) positive semidefinite matrix. We then explore numerous consequences of this main result, including natural applications in the study of quantum resource theories.

From Matrices to Matrix Polynomials

Panayiotis Psarrakos

The study of matrix polynomials of higher degree has attracted considerable attention in recent decades. The interest has been motivated by a wide range of applications of polynomial eigenvalue problems in areas such as differential equations, systems theory, control theory, mechanics and vibrations. In this presentation, we will see how results of the standard matrix theory, concerning pseudospectra, eigenvalue condition numbers, Jordan structure, numerical ranges, nonnegative matrices and normality, have been extended to the setting of matrix polynomials in a natural way. In particular, basic matrix theory can be viewed as the study of the special case of matrix polynomials of first degree.

Unbounded Toeplitz Operators: Invertibility and Riccati Equations
Andre Ran

For a class of unbounded block-Toeplitz operators it will be shown how invertibility is connected to factorization of the symbol, and to existence of a particular solution to an unsymmetric algebraic Riccati equation. This is motivated by a similar result for bounded Toeplitz operators due to Rien Kaashoek, Art Frazho and the speaker. In turn, that result was inspired by a paper by Peter Lancaster, Leiba Rodman and the speaker which largely originated during a three week visit of the speaker in 1985 to Calgary.

This is joint work with Jacob Jaftha (University of Cape Town), Gilbert Groenewald and Sanne ter Horst (both North West University).

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Yamamoto-Nayak's Theorem and its Extension to Lie Groups
Tin Yau Tam

A very recent result of Nayak asserts that $\lim_{m\to\infty} |A^m|^{1/m}$ exists for each $n \times n$ complex matrix A, where $|A| = (A^*A)^{1/2}$, and the limit is given in the language of linear transformation. This is an extension of Yamamoto's result in 1967. We extend the result of Nayak, namely, we prove that $\lim_{m\to\infty} |BA^mC|^{1/m}$ exists for any $n \times n$ complex matrices A, B, and C; the limit is given in matrix language and is independent of B. We then provide generalization in the context of real semisimple Lie groups.

Deflation Strategies for Nonlinear Eigenvalue Problems
Francoise Tisseur

Deflation for linear eigenvalue problems is a standard technique that consists of removing a known eigenvalue or changing it so that the other eigenvalues are easier to find. In this talk we discuss and compare different strategies to deflate eigenvalues of nonlinear eigenvalue problems. We pay particular attention to the quadratic eigenvalue problem, describe a structural engineering application where deflation is needed, and introduce a deflation strategy based on structure preserving transformations.

Regular vs Singular Matrix Polynomials

Fernando de Terán Vergara

A relevant part of Peter's research is devoted to regular matrix pencils and matrix polynomials, and in particular to the *inverse* problem on determining whether some matrix pencil or matrix polynomials exists, with some prescribed eigenstructure and some particular symmetry structure (like self-adjoint, symmetric, or Hermitian). By regular I mean square and with non-identically zero determinant. During (at least) the last two decades, many attention has been also paid, by different researchers (including myself), to singular matrix polynomials, namely matrix polynomials which are either rectangular or square with identically zero determinant. In this talk, I will highlight some relevant differences between regular and singular matrix polynomials, in particular regarding the size of linearizations and the eigenstructure, and I will also review some recent results on the inverse problem about the existence of matrix polynomials with some prescribed eigenstructure. This is based on joint work with F. M. Dopico, D. S. Mackey, P. Van Dooren, J. Prez, and C. Hernando.

Spectral Properties of Token Graphs

Piyush Verma

Let G be a graph on n vertices. For a given integer k such that $1 \leq k \leq n$, the k-token graph $F_k(G)$ of G is defined as the graph whose vertices are the k-subsets of vertex set of G, and two of them are adjacent whenever their symmetric difference is a pair of adjacent vertices in G. We study the structural and spectral properties of token graphs. We describe the adjacency matrix and the Laplacian matrix of $F_k(G)$, and obtain bounds on the adjacency and Laplacian spectral radii of $F_k(G)$. Interestingly, it is found that S_n, the star graph on n vertices has the same Laplacian spectral radius as that of $F_k(S_n)$. It was conjectured that for any graph G, the algebraic connectivity of $F_k(G)$ is equal to the algebraic connectivity of G. This result turned out to be a theorem, as it was proved by using the theory of the continuous Markov chain of random walks and interchange process. However, proving this theorem using algebraic and combinatorial methods is still an open and interesting problem. Using combinatorial techniques, we prove that the theorem holds for a class of graphs that have a cut-vertex of degree $n-1$. We also prove it by restricting the smallest degree of k-token graph of G.

Phase Synchronization and Filters for Quadratic Systems
Ion Zaballa

A real quadratic system $M\ddot{x}(t)+D\dot{x}(t)+Kx(t)=f(t)$, $M,D,K\in\mathbb{R}^{n\times n}$ is said to be *classically damped* if there are real invertible matrices P and Q such that $P(Ms^2+Ds+K)Q=M_ds^2+D_ds+K_d$ with M_d,D_d,K_d diagonal matrices. A necessary condition for a quadratic system to be classically damped is that it must be diagonalizable; that is, there must be unimodular matrices $U(s),V(s)\in\mathbb{R}[s]^{n\times n}$ such that $U(s)(Ms^2+Ds+K)V(s)=M_ds^2+D_ds+K_d$. A necessary and sufficient condition for a quadratic system to be diagonalizable was found in [2] when M is nonsingular and in [4] when $\det M=0$. On the other hand, it is shown in [1,3] that a characterization of the classically damped systems is that the components of their *damped modes of vibration* have the same phase. For vibrating systems this means that the system components perform harmonic motion with the same damped frequency and passing through their equilibrium positions at the same instant of time. Taking this into account a method to decouple (or diagonalize) diagonalizable quadratic systems called *phase synchronization* has been studied in the last years (see [3]). This method consists of introducing time shifts in order to produce a new system whose damped modes of vibration have their components with the same phase. This process generates a decoupling transformation that is real, time-dependent, and generally nonlinear, and it operates in the n-dimensional configuration space spanned by the components of $x(t)$. In practice phase synchronization can be achieved in state-space by using Structure Preserving Transformations on linearizations of the original quadratic system. This approach has the drawback that the linear systems has size twice of the original quadratic system if M is nonsingular. A possible alternative is the use of spectral filters. The goal of this talk is to show how spectral filters can be used to implement the method of phase synchronization working directly on the n-dimensional configuration space. This is joint work with A. Amparan, S. Marcaida (Universidad del País Vasco(UPV/EHU)).

References:

[1] A. Imam F. Ma and M. Morzfeld. The decoupling of damped linear systems in oscillatory free vibration. Journal of Sound and Vibration, 324:408–428, 2009.

[2] P. Lancaster and I. Zaballa. Diagonalizable quadratic eigenvalue problems. Mechanical Systems and Signal Processing, 23(4):1134–1144, 2009.

[3] R. G. Salsa and F. Ma. Advances in the Theory of System Decoupling. Springer, Cham, Switzerland, 2021.

[4] J. C. Zúñiga. Diagonalization of quadratic matrix polynomials. Systems and Control Letters, 59:105–113, 2010.

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