WESTERN CANADA LINEAR ALGEBRA MEETING
Program - Abstracts - Participants

University of Regina

May 27-28, 2023

Organizing Committee

Shaun Fallat, Hadi Kharaghani, Steve Kirkland, Peter Lancaster, Sarah Plosker, Michael Tsatsomeros, Pauline van den Driessche

Local Organizers:
Shaun Fallat, Seyed Ahmad Mojallal, and Prateek Kumar Vishwakarma

Funding

The WCLAM Organising Committee gratefully acknowledges the generous support for this meeting provided by The Pacific Institute for the Mathematical Sciences, the International Linear Algebra Society, the Department of Mathematics & Statistics, and the Faculty of Science at the University of Regina.

Invited Speakers

Jane Breen, Ontario Tech University
Leslie Hogben, Iowa State University

Location

Education Building (Rooms 192, 193), University of Regina

1 Meeting Program (Education Bldg. # 193)

Saturday, May 27, 2023

08:30-09:15 **Registration**

09:15-09:30 **Welcome & Information**

Chair: Steve Kirkland

09:30-10:20 **Jane Breen (ILAS Speaker)**, *Kemeny's constant for Markov chains and random walks on graphs*

10:30-11:00 **Break** *(ED 192)*

Chair: Shaun Fallat

11:00-11:25 **Kerry Ojakian**, *Markov processes and graph labelings*

11:30-11:55 **Parthasarathi Nag**, *Topological insights into class of controllable Linear Time Invariant System [(A,B)]*

12:00-13:30 **Lunch (on your own)**

Chair: Pauline van den Driessche

13:30-13:55 **Rakesh Jana**, *The Bipartite Distance Matrix of a Nonsingular Tree*

14:00-14:25 **Allen Herman**, *Feasibility Conditions for parameters of quotient-polynomial graphs*

14:00-14:25 **Leslie Hogben (invited)**, *Uniform and apportionable matrices*

15:30-16:00 **Break** *(ED 192)*

Chair: Shaun Fallat

16:00-16:25 **Mohsen Zahraei**, *On the generalized numerical ranges of the powers of a matrix*

16:30-16:55 **Avleen Kaur**, *Unraveling the Friedrichs Angle: A Key to Lower Bounds on the Minimum Singular Value*

17:00-17:25 **Seyed Ahmad Mojallal**, *Applications of the vertex-clique incidence matrix of a graph*

18:30 **Informal Banquet–Bushwakker's** *2206 Dewdney Ave.*

Sunday, May 28, 2023

Chair: Michael Tsatsomeros

09:30-09:55	**Colin Garnett**, *Coefficient Support Arbitrary patterns*
10:00-10:25	**Hermie Monterde**, *Twins are mostly sedentary*
10:30-11:00	**Group photo and break** *(ED 192)*

Chair: Hadi Kharaghani

11:00-11:25	**Prateek K. Vishwakarma**, *Inequalities for totally nonnegative matrices: Gantmacher–Krein, Karlin, and Laplace*
11:30-11:55	**Eugene Agyei-Kodie**, *Recursion formulas for determinants of k-Tridiagonal Toeplitz Matrices*
12:00-12:25	**Peter Zizler**, *Singular Value Decomposition at FIFA 2022*
12:30-12:40	**Closing remarks (Fallat)**

Posters (ED 192)

Vlad Zaitsev, *An Introduction to Orthogonal Arrays and Equiangular Lines*

Caleb Van’t Land, *An Attempt to Find a Maximal Set of Equiangular Lines*

2 Abstracts for talks (alphabetical by speaker)

Recursion formulas for determinants of k-Tridiagonal Toeplitz Matrices
Eugene Agyei-Kodie

As defined in [2], we consider an $n \times n$ Toeplitz matrix , T, to be of the form:

$$T_{ij} = \begin{cases} a & ; \quad i = j \\ b & ; \quad |i - j| > k \\ c & ; \quad |i - j| < k \\ 0 & \text{otherwise} \end{cases}$$

There has been a renewed interest in Toeplitz matrices due to their applications in engineering and computational sciences along with their connections to other matrices as well as recent research identifying the role of Toeplitz matrices in Matrix Theory. For instance, one study has shown that any matrix is the product of Toeplitz matrices and another study shows that any square matrix is similar to a Toeplitz matrix. Toeplitz matrices with its spectral properties are of great essence to physics, statistics and signal processing. Moreover, Toeplitz matrices help model problems including computation of spline functions, signal and image processing, polynomial and power series computations etc.

Over the years, there have been studies on Toeplitz matrices such as recursion of determinants of 2-tridiagonal Toeplitz matrix [1] and tridiagonal 2-Toeplitz matrices [3]. In our study, we investigate the determinant of a k-tridiagonal Toeplitz matrices for $k > 2$. By extending the work of Borowska et al.[1] , we identified recursion formulas for determinants of all $k-$ tridiagonals Toeplitz matrices. Thus, we propose to share our findings at the Conference.

References

[1] J. Borowska, L. Łacińska. Eigenvalues of 2-tridiagonal Toeplitz matrix, *Journal of Applied Mathematics and Computational Mechanics,* vol.14, no.14,2015

[2] M. Elouafi, On a relationship between Chebyshev polynomials and Toeplitz determinants,*Applied Mathematics and Computation,* vol.229, pp. 27-33, 2014.

[3] M.J Gover, The eigenproblem of a tridiagonal 2-Toeplitz matrix, *Linear Algebra and its Applications,* vol. 197, pp. 63-78, 1994.

Kemeny's constant for Markov chains and random walks on graphs
Jane Breen

Kemeny's constant is an interesting and useful quantifier of how well-connected the states of a Markov chain are. Though it was first introduced in the 1960s, interest in this concept hasrecently exploded.

This talk will provide an introduction to Markov chains, an overview of the history of Kemeny's constant, discussion of some applications, and a survey of recent results, with an emphasis on thosewhere the combinatorial structure of the Markov chain is of interest. This comes to the forefront when the Markov chain in question is a random walk on a graph, in which case Kemeny's constant is interpreted as a measure of how 'well-connected' the graph is.

Coefficient Support Arbitrary patterns

Colin Garnett

The ideas of spectrally arbitrary patterns and inertially arbitrary patterns have been well studied. We will introduce the concept of coefficient support arbitrary (CSA) patterns that seems to be somewhere in betweenthese two. In particular we are interested in whether a given pattern can attain any zero-nonzero pattern in the coefficients of its characteristic polynomial. CSA patterns can be thought of as a generalization of potentially nilpotent patterns, where ratherthan just a nilpotent realization (i.e. the case where all coefficients of the characteristic polynomial are 0) we consider the case where any number of the coefficients is 0. This concept allows us to harness the power of the computer algebra system Sageto resolve the $2n$ conjecture for n up to 6, and it rules out all but 2 patterns when $n = 7$. Additionally over the real numbers we can describe a pattern as being Coefficient Sign Arbitrary patterns and we can show that this is not equivalent to the concept of spectrally arbitrary for patterns over the real numbers. It seems worthwhile to consider these conditions that can be checked by a computer.

Feasibility Conditions for parameters of quotient-polynomial graphs

Allen Herman

Quotient-polynomial graphs are finite simple graphs whose adjacency algbras are equal to the adjacency algebra of a symmetric association scheme generated by the graph. Strongly-regular graphs and distance-regular graphs are special cases of these graphs. In these two important cases, parameter sets are well-studied because existence of the SRG or DRG requires its parameters to satisfy several feasibility conditions. In this talk I will propose a parameter set scheme for quotient-polynomial graphs and some feasibility conditions that can be checked efficiently.

Uniform and apportionable matrices

Leslie Hogben

There has been extensive study of diagonalization of matrices, or finding the Jordan Canonical Form for a matrix that is not diagonalizable. Diagonalization can be viewed as using a similarity to concentrate the magnitude of all the entries with a small subset of entries. Here we study what can be viewed as reversing this process, spreading out the magnitudes as uniformly as possible. A uniform matrix plays the role of a diagonal matrix in this process. A square complex matrix is *uniform* if all entries have the same absolute value and a square complex matrix is apportionable if it is similar to a uniform matrix. Hadamard matrices and discrete Fourier transforms are important examples of uniform matrices.

Various results and examples are presented. Every rank one matrix is apportionable and there is a procedure to find an apportioning matrix. However, not all spectra and Jordan Canonical Forms are are attainable by uniform matrices. There are expamples of matrices A such at there are infinitely many possible magnitudes of entries of the uniform matrices MAM^{-1}. Gracefully labelled graphs can be used to construct apportionable matrices with prescribed spectra.

Unraveling the Friedrichs Angle: A Key to Lower Bounds on the Minimum Singular Value

Avleen Kaur

Estimating the eigenvalues of a sum of two symmetric matrices, say $P + Q$, in terms of the eigenvalues of P and Q, has a long tradition. To our knowledge, no study has yielded a positive lower bound on the minimum eigenvalue, $\lambda_{\min}(P + Q)$, when $P + Q$ is symmetric positive definite with P and Q singular positive semi-definite. We derive two new lower bounds on $\lambda_{\min}(P + Q)$ in terms of the minimum positive eigenvalues of P and Q. The bounds take into account geometric information by utilizing the Friedrichs angles between certain subspaces. The basic result is when P and Q are two non-zero singular positive semi-definite matrices such that $P + Q$ is non-singular, then $\lambda_{\min}(P + Q) \geq (1 - \cos\theta_F)\min\{\lambda_{\min}(P), \lambda_{\min}(Q)\}$, where $\lambda_{\min}$ represents the minimum positive eigenvalue of the matrix, and θ_F is the Friedrichs angle between the range spaces of P and Q. We will discuss the interaction between the range spaces for some pair of small matrices to elucidate the geometric aspect of these bounds. Such estimates lead to new lower bounds on the minimum singular value of full rank 1×2, 2×1, and 2×2 block matrices in terms of the minimum positive singular value of these blocks. Some examples provided in this talk further highlight the simplicity of applying the results in comparison to some existing lower bounds.

This is joint work with S. H. Lui (Manitoba). Supported by the University of Manitoba Graduate Fellowship (Avleen Kaur) and the Natural Sciences and Engineering Research Council of Canada (S. H. Lui).

Applications of the vertex-clique incidence matrix of a graph

Seyed Ahmad Mojallal

Let G be a graph of order n and let $F = \{C_1, C_2, \ldots, C_k\}$ be an edge clique cover for G. The vertex-clique incidence matrix of G associated with the edge clique cover F is defined as follows:
Corresponding to any edge clique cover F, we define a real $n \times k$ matrix M_F with rows and columns indexed by the vertices in V and the cliques in F, respectively, such that the ij-entry of M_F is real and nonzero if and only if the vertex i belongs to the clique $C_j \in F$. In this talk, we present applications of such matrices in spectral graph theory.

Twins are mostly sedentary

Hermie Monterde

Let A be the adjacency matrix of a graph X. The transmission of quantumstates within a quantum spin network represented by the graph X is determined by the unitary matrix $U(t) = exp(itA)$. In fact, one may interpret $|U(t)_{u,v}|^2$ as the probability of quantum state transfer between vertices u and v in X. Whilesome pairs of twin vertices (vertices that share the same neighbours) exhibit accurate transmission of quantum states within a graph, we will show in this talk that vertices in a set of twins of size at least three resist high probability quantum state transfer

Topological insights into class of controllable Linear Time Invariant System [(A,B)]

Parthasarathi Nag

Consider a linear time invariant control system

$$\frac{dx}{dt} \;=\; Ax + Bu,\; x(0) = 0 \quad (1)$$

where $A : \mathcal{X} \to \mathcal{X}$ is a linear map such that $\mathcal{X} \cong \mathbb{C}^n$, $B : \mathcal{U} \to \mathcal{X}$ is a linear map such that $\mathcal{U} \cong \mathbb{C}^m$, the state of the system $x \in \mathcal{X}$ and the control input $u \in \mathcal{U}$. A Laplace transformed of the control system (1) can be rewritten as

$$sX = AX \;+\; BU \quad (2)$$

where $X = X(s)$, $U = U(s)$ and $s \in \mathbb{C}$. Hermann and Martin constructed vector bundles on the Riemann sphere defined by linear time invariant control system (2) in 1978. In that same article Hermann and Martin mentions that **"it can be proved, . . . that the Chern numbers of these line bundles are equal to Kronecker indices."**

The current research presentation is motivated by the above statement. We are going to show that the first Chern number of rank m Hermann-Martin bundle for a controllable class of system given by (1) or (2) such that

$$rank[B \; AB \; \cdots \; A^{n-1}B] = n$$

and $rank\ B = m$ and the Brunovsky canonical form can be obtained by the first Chern numbers, via Brikhoff-Grothendeick decomposition, of the Hermann-Martin bundle. These first Chern numbers are essentially the Kronecker indices. This article attempts to provide some topological insights into controllable linear time invariant systems.

Markov processes and graph labelings

Kerry Ojakian

We motivate various graph labeling questions by their connection to questions about Markov processes. We consider a basic Markov process on an n vertex graph, i.e. an entity moves randomly through the graph, according to proscribed edge probabilities. For each vertex, we consider the expected proportion of time the object will be there, and can thus associate a length n probability vector with the Markov process; i.e. its stationary vector. Our first fundamental question is this: Given a graph, which stationary vectors can be achieved by adjusting the edge probabilities of the Markov process? There is a nice condition for determining which vectors are possible (the answer is essentially buried in the literature). For the main part of our investigation, we consider the "rank vector" derived naturally from the stationary vector; for example, if the stationary vector is [5, 9, 8, 7], then its corresponding rank vector is [1, 4, 3, 2]. Our second fundamental question is this: Given a graph, by adjusting the probabilities of the Markov process, what rank vectors are achievable? Using standard Markov theory we reduce this question to a graph labeling question. Given an edge labeling, we can determine a length n vector by assigning each vertex the sum of the numbers on its incident edges, then from this vector we derive a rank vector. We ask: Which rank vectors are achievable by such an edge labeling? The graph labeling question turns out to be equivalent to our second fundamental question, so we focus on this graph labeling question. We answer the question in special cases, and give a nice condition which we conjecture to fully answer the question. Behind the scenes, these questions are motivated by an interest in associating centrality measures (i.e. betweeness centrality, closeness centrality, etc) to natural graph processes (such as a Markov process). This is joint work with David Offner.

The Bipartite Distance Matrix of a Nonsingular Tree

Rakesh Jana

Abstract The bipartite adjacency matrix (J. A. Bondy and U. S. R. Murty. *Graph theory*, Springer-Verlag London, New York, 2008) is used to store the adjacency information for a bipartite graph. Similar to the bipartite adjacency matrix, we define the *bipartite distance matrix* of a bipartite graph. That is, the *bipartite distance matrix* $\mathfrak{B}(G)$ of a bipartite graph G with $m+n$ vertices is a $m \times n$ matrix whose (i, j)th entry is the distance between vertices l_i and r_j, where $L := \{l_1, \ldots, l_m\}$, $R := \{r_1, \ldots, r_n\}$ is a vertex bipartition of G. If $|L| = |R| = p$, then $\det \mathfrak{B}(G)$ is always a multiple of 2^{p-1}. Based on this observation, we define the *bipartite distance index* of G as $\texttt{bd}(G) := \det \mathfrak{B}(G)/(-2)^{p-1}$.

We use the word 'nonsingular tree' to mean a tree with a (unique) perfect matching. We show that the bipartite distance index of a nonsingular tree T satisfies an interesting inclusion-exclusion type of principle at any matching edge of the tree. Even more interestingly, we show that the bipartite distance index of a nonsingular tree T can be completely characterized by the structure of T via what we call the f-alternating sums.

Quite similar to Graham, Hoffman and Hosoya (*On the distance matrix of a directed graph*, Journal of Graph Theory, 1(1):85–88, 1977) result on the distance matrix, we identify some basic elements and a merging operation and show that each of the trees that can be constructed from a given set of elements, sequentially using this operation, have the same bipartite distance index, independent of the order in which the sequence is followed.

The talk is based on following papers.

1. RB Bapat, Rakesh Jana, and S Pati. The bipartite distance matrix of a nonsingular

tree. *Linear Algebra and its Application*, 631:254-281, 2021.

2. Rakesh Jana. A q-analogue of the bipartite distance matrix of a nonsingular tree, *Discrete Mathematics*, 346(1):113153, 2023.

Inequalities for totally nonnegative matrices: Gantmacher–Krein, Karlin, and Laplace

Prateek Kumar Vishwakarma

Set theoretic operations preserving total nonnegativity naturally translate into operations preserving determinantal inequalities for this class of matrices. We introduce set row/column operations that act directly on all determinantal inequalities for totally nonnegative matrices, and yield inequalities for these matrices. These operations assist in discovering additive inequalities for totally nonnegative matrices embedded in classical identities due to Laplace (1772) and Karlin (1968). This as well generalizes the seminal inequalities due to Gantmacher–Krein (1941) over these matrices. These refinements reveal sequences of inequalities oscillating about zero. The proposed set operations and planar networks corresponding to totally nonnegative matrices facilitate obtaining a novel class of oscillating inequalities. Furthermore, it is needless to say that these set row/column operations birth an algorithm that can detect certain determinantal expressions that do not form an inequality over totally nonnegative matrices. However, the algorithm completely characterizes inequalities comparing products of pairs of minors. Moreover, the underlying row/column operations add that these inequalities are offshoots of the complementary ones. These novel results seem very natural, which in addition thoroughly describe and enrich the classification due to Fallat–Gekhtman–Johnson [*Adv. Appl. Math.* 2003] and later Skandera [*J. Algebr. Comb.* 2004]. *This is joint work with S. Fallat.*

On the generalized numerical ranges of the powers of a matrix

Mohsen Zahraei

In the presennt paper, we provide several inequalities for the generalized numerical radius of the powers of the matrices as introduced by Charles R. Johnson and I. M. Spitkovsky in [1]. In addition, we added some new comments including generalization of [2, Theorem 2.1] (and subsequent results) to the setting of bounded operators $B(\mathcal{H})$ on a Hilbert space $\mathcal{H}$ and also some of the notions that appear in [2, Theorem 2.1] are well defined in this more general setting.

[1] C. R. Johnson and I. M. Spitkovsky, Inequalities Involving the Numerical Radius, Linear and Multilinear Algebra, **37** (1994), 13-24.

[2] H.-L. Gau, Kuo-Zhong Wang, Pei Yuan Wu *Equality of Numerical Ranges of Matrix Powers*, Linear Algebra and its Applications **578** (2019), 95-110.

Singular Value Decomposition at FIFA 2022

Peter Zizler

Team's offense and defense ability can be estimated by goalcounts, for and against, in round robin tournaments. We introduce of-fense and defense scores for the teams, stemming out of the singular value decomposition of a matrix, that are sensitive to goal scoring on good orbad defensive teams, and goals against from good or bad offense teams.Further finer analysis of team interactions in a tournament is discussed along with numerical considerations.

3 Other Relevant Conference Information:

- University Map: Available on the conference website.
- Parking is available free of charge on the weekends (avoid parkades and accessbility spots) Lots 2,6, and 10 will work for visitors.
- Dining: There is very limited dining on campus over the weeked. Luther Colloege Cafteria is open on Saturday and Sunday. There are also some restaurants available in a strip mall at the corner of Kramer Bvld and Wascana Parkway (5-8 minute walk).
- Group Photo: Will take place on Sunday during the morning break...
- Zoom Link: https://uregina-ca.zoom.us/j/4941174436 (Meeting ID: 494 117 4436)

4 Participants

Eugene Agyei-Kodie, University of Texas at El Paso, eagyeikodi@miners.utep.edu
Matthew Alexander, University of Regina, alexand2tice@gmail.com
Jane Breen, Ontario Tech University, Jane.Breen@ontariotechu.ca
Laune Burns, University of Regina, burns24l@uregina.ca
Sarah Carnochan Naqvi, University of Regina, Sarah.Carnochan.Naqvi@uregina.ca
Shane Crerar, University of Regina, crerars@uregina.ca
Shaun Fallat, University of Regina, shaun.fallat@uregina.ca
Colin Garnett, Black Hills State University, Colin.Garnett@bhsu.edu
Amitabh Halder, University of Regina, akh581@uregina.ca
Allen Herman, University of Regina, Allen.Herman@uregina.ca
Leslie Hogben Iowa State University, hogben@aimath.org
Saied Hooshyar, University of Regina, shn936@uregina.ca
Chun-Hua Guo, University of Regina, Chun-Hua.Guo@uregina.ca
Rakesh Jana, Indian Institute of Technology Bombay, jana.rakesh.math@gmail.com
Hadi Kharaghani, University of Lethbridge, hadi.kharaghani@gmail.com
Steve Kirkland, University of Manitoba, Stephen.Kirkland@umanitoba.ca
Avleen Kaur, University of Saskatchewan, avleen.kaur@usask.ca
Peter Lancaster, University of Calgary, lancaste@ucalgary.ca
Karen Meagher, University of Regina, Karen.Meagher@uregina.ca
Seyed Ahmad Mojallal, University of Regina, ahmad_mojalal@yahoo.com
Hermie Monterde, University of Manitoba, monterdh@myumanitoba.ca
Parthasarathi Nag, Black Hills State University, P.Nag@bhsu.edu
Kerry Ojakian, Bronx Community College, kerryojakian@gmail.com
Avner Sadikov, University of Regina, avner.sadikov@hotmail.com
Yash Shamsundar, Indian Institute of Science Education and Research (IISER) Bhopal & University of Lethbridge, yashk19@iiserb.ac.in
Fernando Szechtman, University of Regina, fernando.szechtman@gmail.com
Michael Tsatsomeros, Washington State University, tsat@wsu.edu
Christopher van Bommel, University of Guelph, cvanbomm@uoguelph.ca
Pauline van den Driessche, University of Victoria, vandendr@uvic.ca
Caleb Van't Land, University of Lethbridge, caleb.vantland@uleth.ca
Prateek K. Vishwakarma, University of Regina, Prateek.Vishwakarma@uregina.ca
Peter Wadel, University of Regina, wadel20p@uregina.ca
Jeffery Xu, University of Regina, jxd937@uregina.ca
Boting Yang, University of Regina, Boting.Yang@uregina.ca
Mohsen Zahraei, Islamic Azad Universtiy & University of Regina, mzahraei326@gmail.com
Vlad Zaitsev, Univerity of Lethbridge, vlad.zaitsev@uleth.ca
Sandra Zilles, University of Regina, zandra.zilles@gmail.com
Peter Zizler, Mount Royal University, pzizler@mtroyal.ca