Prairie Discrete Math Workshop

Conference Program

June 9 & 10, 2022

Online / University of Regina

https://uregina-ca.zoom.us/j/98675887630
Meeting ID: 986 7588 7630

Conference Sponsors

The Pacific Institute for the Mathematical Sciences (PIMS)

Conference Organizers

- Karen Gunderson, Assistant Professor, University of Manitoba
- Karen Meagher, Professor, University of Regina
- Hermie Monterde, PhD Student, University of Manitoba
- Joy Morris, Professor, University of Lethbridge

Schedule

Times given in local Saskatchewan time (that is Mountain Daylight Saving Time)

Time	June 9	June 10
8:00am	**Invited Speaker** Mateja Šajna	**Invited Speaker** Karen Gunderson
9:00am	**3 minute talks** (round 1)	**3 minute talks** (round 2)
9:30 am	Break	Break
10:00am	**Contributed Talks** Dennis Kinoti Gikunda Christopher van Bommel Xiaohong Zhang	**Contributed Talks** Andrii Arman Kristaps Balodis
11:00am	**Invited Speaker** Svenja Huntemann	**Invited Speaker** JD Nir
noon	Lunch	Lunch
1:30pm	**Contributed Talks** Ahmad Mojallal Prateek Kumar Vishwakarma Sooyeong Kim	**Contributed Talks** Neha Joshi Ferdinand Ihringer Davoud Abdi
2:30pm	Break	Break Conference Photo
3:00pm	**Invited Speaker** Boting Yang	**Invited Speaker** Venkata Raghu Tej Pantangi

Invited Speakers

- **Karen Gunderson**
  University of Manitoba
  Turán numbers and tournament switching
- **Svenja Huntemann**
  Concordia University of Edmonton
  Enumeration of Positions in Placement Games
- **JD Nir**
  University of Manitoba
  Close Enough! How to (Probably) Calculate the Chromatic Number
- **Mateja Šajna**
  University of Ottawa
  The Surprising Honeymoon Oberwolfach Problem
- **Venkata Raghu Tej Pantangi**
  University of Lethbridge
  Erdős-Ko-Rado Module Property
- **Boting Yang**
  University of Regina
  Computing the One-Visibility Cop Number and Strategies for Trees

Lightning Talks Schedule

Time	June 9	June 10
9:00-9:03	Hermie Monterde	Michael Cavers
9:04-9:07	Allen Herman	Ben Cameron
9:08-9:11	Varsha Singh	Gary Au
9:12-9:15	Ferdinand Ihringer	Sooyeong Kim
9:16-9:19	Shaun Fallat	~~Karen Meagher~~ Robert Craigen

Aims and Scope

The main objective of the Prairie Discrete Mathematics Workshop (PDMW) is to bring together researchers in discrete mathematics in the prairie region (Manitoba, Saskatchewan and Alberta), as well as neighbouring provinces and states, with the goal of providing opportunities for networking and joint research.

Various forms of the PDMW have been held since 1995, and the first workshop in its current form was at the University of Regina in 2003. Since then it has been held annually (except for 2007) at universities across the prairies, and is returning to Regina for the first time since 2003.

For 2022, the workshop will cover a range of different areas of discrete mathematics and related areas of computer science. We have invited six speakers from the prairie region, whose interests include graph theory, design theory, applied combinatorial enumeration, combinatorial algorithms, and computational learning theory.

Regina Land Acknowledgement

The University of Regina is situated on Treaty 4 lands with a presence in Treaty 6. These are the territories of the nêhiyawak, Anihšnāpēk, Dakota, Lakota, and Nakoda, and the homeland of the Métis/Michif Nation. Today, these lands continue to be the shared Territory of many diverse peoples from near and far.

List of Speakers and Abstracts

Mateja Šajna

University of Ottawa

The Surprising Honeymoon Oberwolfach Problem

The Honeymoon Oberwolfach Problem is a surprisingly interesting variation on the spouse-avoiding variant of the Oberwolfach Problem. As a scheduling problem, HOP$(2m_1, \ldots, 2m_t)$ asks whether is it possible to arrange $n = m_1 + \ldots + m_t$ couples at a conference at t round tables of sizes $2m_1, \ldots, 2m_t$ for $2n - 2$ meals so that each participant sits next to their spouse at every meal, and sits next to every other participant exactly once. In graph-theoretic terms, a solution to HOP$(2m_1, \ldots, 2m_t)$ is a decomposition of $K_{2n} + (2n-3)I$, the complete graph on $2n$ vertices with $2n-3$ additional copies of a chosen 1-factor I, into 2-factors, each consisting of disjoint I-alternating cycles of lengths $2m_1, \ldots, 2m_t$. It is also equivalent to a semi-uniform 1-factorization of K_{2n} of type $(2m_1, \ldots, 2m_t)$. Thus, the Honeymoon Oberwolfach Problem is related not only to the famous Oberwolfach Problem, but also to Kotzig's conjecture on perfect 1-factorizations.

I will present several results, most notably, a complete solution to the case with uniform cycle lengths. This is joint work with my students Dene Lepine and Mary Rose Jerade.

Dennis Kinoti Gikunda

Kenyatta University

Algorithm Analysis for Big Data

Time efficiency is important in deciding which algorithm to use, but it is not the only factor to consider. The amount of memory space required is also important, and there are mathematical techniques for estimating space efficiency, just as there are for estimating time efficiency. These mathematical techniques are founded on understanding of functions, combinatorics, and recurrence relations. Furthermore, Paul Bachmann's O-notation and Donald Knuth's Ω and Θ notations provide approximations that make evaluating large scale differences in algorithm efficiency simple. Big data carries the burden of parallel data processing, necessitating the modification of current algorithm analysis to meet the needs of the new technology.

Key Words: Sequences, Algorithm Analysis, Big Data, Technology

Christopher van Bommel

University of Manitoba

Investigating Perfect State Transfer on Trees

Quantum computing is believed to provide many advantages over traditional computing, particularly considering the speed at which computations can be performed. One of the challenges that needs to be resolved in order to construct a quantum computer is the transmission of information from one part of the computer to another. This transmission can be implemented by spin chains, which can be modeled as a graph, and analyzed using algebraic graph theory. We investigate the possibility of perfect state transfer on trees and discuss constructions in which it is impossible.

Xiaohong Zhang

University Waterloo

Oriented Cayley graphs with special eigenvalues

Let G be a finite abelian group. An oriented Cayley graph on G is a Cayley digraph $X(G, C)$ such that C and $-C$ are disjoint. Consider the $(0, 1, -1)$ skew-symmetric adjacency matrix of an oriented Cayley graph $X = X(G, C)$. We give a characterization of when all the eigenvalues of X are integer multiples of $\sqrt{\Delta}$ for some square-free integer $\Delta < 0$. This also characterizes oriented Cayley graphs on which the continuous quantum walks are periodic, a necessary condition for the walk to admit uniform mixing or perfect state transfer.

Svenja Huntemann

Concordia University of Edmonton

Enumeration of Positions in Placement Games

Placement games are two-player games played on finite graphs in which the players take turns placing tokens, without moving or removing them later. Many of these games have been studied using tools from combinatorial game theory, the study of perfect information games, but few are completely solved. A relatively new topic of interest in combinatorial game theory is the enumeration of specific types of positions. The number of positions of a certain type in relation to all positions gives an indication of the complexity of analysis and which analysis tool might be most efficient. I will discuss two projects on enumeration of positions in placement games based on the number of pieces placed by each player. First, the game Domineering is played on a grid, or subgraph of a grid, and the players place dominoes (tokens on two adjacent vertices), with one playing horizontally, the other vertically. For this game, a technique of tiling and matrix multiplication can be used to enumerate all positions or just specific positions towards the end of the game. Secondly, in many placement games the placement of tokens only depends on the distance to previously placed tokens, such as in the games COL and SNORT. We have found generating functions for many of these games played on a variety of graphs and pose several conjectures for others. This is joint work with Neil McKay and Lexi Nash.

Ahmad Mojallal

University of Regina

Open Problem on σ-invariant

Let G be a graph of order n with m edges. Also let $\mu_1 \geq \mu_2 \geq \cdots \geq \mu_{n-1} \geq \mu_n = 0$ be the Laplacian eigenvalues of graph G and let $\sigma = \sigma(G)$ $(1 \leq \sigma \leq n)$ be the largest positive integer such that $\mu_\sigma \geq \frac{2m}{n}$. In this talk, we show that $\mu_2(G) \geq \frac{2m}{n}$ for almost all graphs. Moreover, we characterize the extremal graphs for any graphs. We also provide the answer to Problem 3 in [Distribution of Laplacian eigenvalues of graphs, Linear Algebra Appl. 508 (2016), 48–61], that is, the characterization of all graphs with $\sigma = 1$. Finally, we present a few relations between σ and other graph invariants.

This is joint work with Kinkar Ch. das, Sungkyunkwan University

Prateek Kumar Vishwakarma

University of Regina

A Gantmacher–Krein determinantal inequality via planar networks

Gantmacher and Krein discovered a relation between the determinant of a totally nonnegative matrix and its partial Laplace expansions along the first row using Sylvester's determinant identity. We shall present an alternate proof of the same using the re-parameterization of totally nonnegative matrices in terms of weighted acyclic directed planar networks, which is popularly known as the converse of Lindström's lemma. To conclude, we shall articulate some of the outcomes of employing this method in related and ongoing work with Shaun Fallat.

Sooyeong Kim

Università di Pisa

Families of graphs with twin pendent paths and the Braess edge

In the context of a random walk on an undirected graph, Kemeny's constant can measure the average travel time for a random walk between two randomly chosen vertices. My interest is in graphs that behave counter-intuitively in regard to Kemeny's constant. In particular, I present graphs with a cut-vertex at which at least two branches are paths, regarding whether the insertion of a particular edge into a graph results in an increase of Kemeny?s constant. I provide several tools for identifying such an edge in a family of graphs and for analyzing asymptotic behaviour of the family regarding the tendency to have that edge; and classes of particular graphs are given as examples. Furthermore, asymptotic behaviours of families of trees are described.

Boting Yang

University of Regina

Computing the One-Visibility Cop Number and Strategies for Trees

In this talk, we consider the one-visibility cops and robbers game. We discuss cop-win strategies for searching trees. We give a linear-time algorithm for computing the one-visibility cop number of trees. We also present an $O(n \log n)$-time algorithm for computing the cop-win strategies of trees, where n is the number of vertices.

Karen Gunderson

University of Manitoba

Turán numbers and tournament switching

The *Turán number* for a fixed r-uniform hypergraph $\mathcal{H}$ is the maximum number of hyperedges in any r-uniform hypergraph on n vertices containing no copy of $\mathcal{H}$. I will discuss some exact Turán numbers that are obtained from a 'switching operation' on tournaments. Two tournaments on the same vertex set are *switching equivalent* if one can be obtained from the other by interchanging all edges between two disjoint sets that partition the vertices. In some cases, infinite classes of extremal hypergraphs can be constructed by taking as hyperedges all sub-tournaments in a fixed switching equivalence class. The target equivalence class can either be defined using some special classes of tournaments with connections to design theory or by computer search. I will discuss some uniqueness results for the extremal results arising from algebraically-defined tournaments, and some possible avenues for further research directions. Based on joint work with Jason Semeraro.

Andrii Arman

University of Manitoba

Upper bounds on the chromatic number of n-dimensional Euclidean space

The chromatic number of the n-dimensional Euclidean space $\mathbb{R}^n$ is the least number of colors needed to color the points of the space so that every two points distance one apart receive different colors.

In this talk I will present new upper bounds for the chromatic number of $\mathbb{R}^n$ in low dimensions ($5 \leq n \leq 38$). The talk is based on a join work with Andriy Bondarenko, Andriy Prymak, and Danylo Radchenko.

Kristaps Balodis

University of Calgary

On the cycle representation of graphs

Given a graph G, there exists a natural action of its automorphism group on its cycle space. In this talk we explore several basic examples and speculate on the ways these representations might encode graph theoretic information.

JD Nir

University of Manitoba

Close Enough! How to (Probably) Calculate the Chromatic Number

Determining the chromatic number of a graph is an **NP**-complete problem. Surprisingly, though, "most" graphs that are "similar" have the same chromatic number. To formalize this notion, we study the chromatic number of various models of random graphs. This problem boasts over seventy years of clever tricks, not only from probability and graph theory but also linear algebra, complex analysis, and even statistical physics. In this talk, we'll look at how these methods were used in historic breakthroughs as well as in recent results I've published on directed graphs (with Karen Gunderson) and random lifts (with Xavier Pérez-Giménez).

Neha Joshi

University of Regina

Fusions of the generalized Hamming scheme on a strongly regular graph

We say $\mathcal{B}$ is a fusion of an association scheme $\mathcal{A}$, if it is an association scheme where each basis element of $\mathcal{B}$ is a union of basis elements of $\mathcal{A}$. One of the most important example of an association scheme for coding theory is the Hamming scheme, $H(n, q)$. Suppose $\mathcal{A} = \{A_0, A_1, A_2\}$ be a rank 3 association scheme and both A_1 and A_2 are adjacency matrices of strongly regular graphs. The generalized Hamming scheme

$$H(2,\mathcal{A}) = \{A_0 \otimes A_0,\ A_1 \otimes A_1,\ A_2 \otimes A_2,\ (A_0 \otimes A_1) + (A_1 \otimes A_0),\\ (A_0 \otimes A_2) + (A_2 \otimes A_0), (A_1 \otimes A_2) + (A_2 \otimes A_1)\}$$

is one of the fusions of the rank 9 association scheme, $\mathcal{A} \otimes \mathcal{A}$. In this talk, we determine the parameters of all strongly regular graphs for which the generalized Hamming scheme has extra fusions in addition to the one arising from the trivial fusion of $\mathcal{A}$. We also show that for any fusion $\mathcal{B}$ of $\mathcal{A}$, the generalized Hamming scheme $H(n,\mathcal{B})$ is a nontrivial fusion of $H(n,\mathcal{A})$.

Ferdinand Ihringer

Universiteit Gent

The Density of Complementary Subspace

Let V be a finite vector space of dimension $d = e + e'$ over the field with q elements. Consider a family Y_1 of e-spaces and a family Y' of e'-spaces with positive density of at least α each. We show, using an easy argument relying on the expander mixing lemma and well-known properties of the irreducible modules of Grassmann graphs, that the probablity of $S_1 \cap S_2 = \{0\}$ for $(S_1, S_2) \in Y_1 \times Y_2$ is at least $\omega_q(e)\left(1 - \frac{1-\alpha}{\alpha} q^{-\frac{d}{2}}\right)$, where $\omega_q(e) = \prod_{i=1}^{e}(1 - q^{-i})$.

Our motivation is as follows: Suppose that V is equipped with a nondegenerate reflexive sesquilinear form σ. Let Y_1 and Y_2 be the families of nondegenerate subspaces with respect to σ. Using long and sophisticated geometric arguments it is shown in [1] that the probability of $S_1 \cap S_2 = \{0\}$ is at least $1 - Cq^{-1}$ for relatively small C, while leaving a few cases open. Our linear algebra technique takes care of the open cases in [1], slightly improves C, and avoids any deep dives into geometric arguments.

This is joint work with Stephen Glasby (University of Western Australia) and Sam Mattheus (Vrije Universiteit Brussel).

[1] S. P. Glasby, A. C. Niemeyer, C. E. Praeger, The probability of spanning a classical space by two non-degenerate subspaces of complementary dimensions, *arXiv*:2109.10015v1 (2021).

Davoud Abdi

University of Calgary

Siblings of Countable NE-free Posets

Two structures R and S are *equimorphic* when each embeds in the other; we may also say that one is a *sibling* of the other. Generally, it is not the case that equimorphic structures are necessarily isomorphic: the rational numbers, considered as a linear order, has up to isomorphism, continuum many siblings. Let $Sib(R)$ be the number of siblings of R, these siblings are counted up to isomorphism. Thomassé conjectured that for each countable relational structure R, made of at most countably many relations, $Sib(R) = 1$, $\aleph_0$ or $2^{\aleph_0}$. There is an alternative case of interest, namely whether $Sib(R) = 1$ or infinite for a relational structure R of any cardinality.

In this talk, I will introduce NE-free posets, classify them and give a sketch of proof of the alternative Thomassé conjecture for countable NE-free posets.

Venkata Raghu Tej Pantangi

University of Lethbridge

Erdős-Ko-Rado Module Property

The classical Erdős-Ko-Rado (EKR) theorem and its variants can be translated into characterizing maximum co-cliques of graphs in Association schemes. In this talk, we will focus on EKR type theorems in permutation groups. Let G be a finite group acting transitively on X. We wish to characterize the maximum co-cliques in the derangement graph $\Gamma(G; X)$. This is the graph whose vertex set is the group G, with (g, h) being an edge if and only if gh^{-1} does not fix any point in X. Cosets of point stabilizers are canonical examples of intersecting sets. A group action is said to satisfy the EKR property if the size of every intersecting set is bounded above by the size of a point stabilizer. A group action is said to satisfy the strict-EKR property if every maximum intersecting set is a coset of a point stabilizer. It is an active line of research to find group actions satisfying these properties. It was shown that all 2-transitive satisfy the EKR property. While some 2-transitive groups satisfy the strict-EKR property, not all of them do. However a recent result shows that all 2-transitive groups satisfy the slightly weaker "EKR-module property"(EKRM), that is, the characteristic vector of a maximum intersecting set is a linear span of characteristic vectors of cosets of point stabilizers. We will discuss about a few more infinite classes of group actions that satisfy the EKRM property. We will also discuss a characterization of the EKRM property using characters of G. We will also discuss EKRM property in the context of a few other variants of the EKR problem.

Conference Participants

Ahmad Mojallal	University of Regina
Albert Artiles	University of Washington
Allen Herman	University of Regina
Andriaherimanana Sarobidy Razafimahatratra	University of Regina
Andrii Arman	University of Manitoba
Antonina P. Khramova	Eindhoven University of Technology
Baruch Sokaribo	University of Regina
Belindar Atieno Juma	Kenyatta University
Ben Cameron	King's College
Bills Sands	University of Calgary
Boting Yang	University of Regina
Chi Hoi Yip	University of British Columbia
Christopher van Bommel	University Manitoba
Cody Antal	University of Regina
Congjie Shi	York University
Davoud Abdi	University of Calgary
Dennis Kinoti Gikunda	Kenyatta University
Etienne Marceau	Université Laval
Ferdinand Ihringer	Universiteit Gent
Gary Au	University of Saskatchewan
Hao Sun	University of Waterloo
Henry Garrett	Independent
Hermie Monterde	University of Manitoba
Hritik Punj	University of Manitoba
Jared Gobin	University of Manitoba
JD Nir	University of Manitoba

Joy Morris	University of Lethbridge
Karen Gunderson	University of Manitoba
Kristaps Balodis	University of Calgary
Lord Kavi	University of Ottawa
Mahsa Shirazi	University of Regina
Margaret Esther Cruz	University of the Philippines - Diliman
Mateja Šajna	University of Ottawa
Max Gutkin	University of Manitoba
Melvin Adekanye	The King's University
Michael Cavers	University of Toronto Scarborough
Mitra Maleki	University of Regina
Neha Joshi	University of Regina
Prateek Kumar Vishwakarma	University of Regina
Rachana Soni	Bennett University
Rachel Evans	University of Regina
Robert Craigen	University of Manitoba
Ruiyang Chen	University of Manitoba
Shaun Fallat	University of Regina
Sooyeong Kim	Università di Pisa
Svenja Huntemann	Concordia University of Edmonton
Venkata Raghu Tej Pantangi	University of Lethbridge
Varsha Singh	IIT Jodhpur Rajasthan India
Wayne Broughton	UBC Okanagan
William Kellough	University of Manitoba
Winifred Mutuku	Kenyatta University
Xiaohong Zhang	University of Waterloo

Pacific Institute for the Mathematical Sciences

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        ii. The PIMS Director: director@pims.math.ca
        iii. The PIMS COO: Denise@pims.math.ca
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Revised: October, 2021