An Gentle Introduction to Erdős-Ko-Rado Combinatorics

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Slides available at: https://uregina.ca/ meagherk/NZMS.pdf

How to Test your Retro Pacman Game

There are several “parameters”:

1. **PacMan’s speed:** SLOW FAST
2. **Ghosts’ image:**

normal

blue

3. **Maze:**

α

β

4. **Fruit:**

bananas

cherries

To test all of these parameters, make a chart

	speed	ghosts	Maze type	fruit
test 1	slow		α	
test 2	slow		α	
test 3	slow		β	
test 4	slow		β	
test 5	slow		α	
test 6	slow		α	
test 7	slow		β	
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

There are $2 \times 2 \times 2 \times 2 = 16$ different combinations!

Karen’s Solution to get to an Early Lunch

With these 5 tests, for any two parameters I will test all possible 4 combinations:

	PacMan’s Speed	Ghost’s Image	Maze	Fruit on Screen
test 1:	slow		α	
test 2:	slow		β	
test 3:	fast		β	
test 4:	fast		α	
test 5:	fast		β	

Karen’s Solution to get to an Early Lunch

	PacMan’s Speed	Ghost’s Image	Maze	Fruit on Screen
test 1:	slow (1)	 (1)	α (1)	 (1)
test 2:	slow (1)	 (0)	β (0)	 (0)
test 3:	fast (0)	 (1)	β (0)	 (0)
test 4:	fast (0)	 (0)	α (1)	 (0)
test 5:	fast (0)	 (0)	β (0)	 (1)
	$\{1,2\}$	$\{1,3\}$	$\{1,4\}$	$\{1,5\}$

This will be be an effective test if the system has the properties

1. any two intersect makes the pair (1 1)
2. the sets are smaller than half; makes the pair (0 0)
3. distinct, and the same size; makes the pairs (0 1) and (1 0)

My focus is on how to build the sets systems.

Set systems

Definition

1. A k**-set system** is a collection of subsets of $\{1, 2, \ldots, n\}$ each subset has size k.
2. A k-set system is **intersecting** if for all sets A, B in the system $A \cap B \neq \emptyset$.
3. A k-set system is t**- intersecting** if for all sets A, B in the system $|A \cap B| \geq t$.

Example (An intersecting family with $k = 3$ and $n = 7$ of size 13)

$$\{\mathbf{1}, \mathbf{2}, \mathbf{3}\}, \{\mathbf{1}, \mathbf{2}, 4\}, \{\mathbf{1}, \mathbf{2}, 5\}, \{\mathbf{1}, \mathbf{2}, 6\}, \{\mathbf{1}, \mathbf{2}, 7\},$$
$$\{\mathbf{1}, \mathbf{3}, 4\}, \{\mathbf{1}, \mathbf{3}, 5\}, \{\mathbf{1}, \mathbf{3}, 6\}, \{\mathbf{1}, \mathbf{3}, 7\}, \{\mathbf{2}, \mathbf{3}, 4\},$$
$$\{\mathbf{2}, \mathbf{3}, 5\}, \{\mathbf{2}, \mathbf{3}, 6\}, \{\mathbf{2}, \mathbf{3}, 7\}$$

Every set has at least 2 elements from $\{1, 2, 3\}$ with size $\binom{n-3}{k-3} + \binom{3}{2}\binom{n-3}{k-2}$.

For k, t and n what is the largest intersecting set system?

Examples

Definition

A **canonical** t-intersecting k-set system is the collection of all k-subsets that contain a fixed t-set. A **canonical** t-intersecting k-set system has size

$$\binom{n-t}{k-t}$$

.

Example (A canonical intersecting family with $k = 3$ and $n = 7$ of size 15)

$$\{\mathbf{1},2,3\}, \{\mathbf{1},2,4\}, \{\mathbf{1},2,5\}, \{\mathbf{1},2,6\}, \{\mathbf{1},2,7\},$$
$$\{\mathbf{1},3,4\}, \{\mathbf{1},3,5\}, \{\mathbf{1},3,6\}, \{\mathbf{1},3,7\}, \{\mathbf{1},4,5\},$$
$$\{\mathbf{1},4,6\}, \{\mathbf{1},4,7\}, \{\mathbf{1},5,6\}, \{\mathbf{1},5,7\}, \{\mathbf{1},6,7\}$$

Every set contains 1.

This is bigger than the previous set.

The Erdős-Ko-Rado Theorem

Theorem (Erdős-Ko-Rado Theorem for t-intersecting sets, 1961 EKR)

Let $\mathcal{A}$ be a t-intersecting k-set system on an n-set. If $n > f(k,t)$, then

1. $|\mathcal{A}| \leq \binom{n-t}{k-t}$,
2. *$\mathcal{A}$ meets this bound if and only if it is canonically t-intersecting.*

Theorem (Erdős-Ko-Rado Theorem—Simplified)

Let $\mathcal{A}$ be an intersecting k-set system on an n-set. If $n \geq 2k+1$, then

1. $|\mathcal{A}| \leq \binom{n-1}{k-1}$,
2. *$\mathcal{A}$ meets this bound if and only if it is canonically intersecting.*

Quotes about Erdős-Ko-Rado Theorems

> *'While the notion of an intersecting set system is rather dry on the face of things, it applies broadly to many areas of mathematics, as sets are virtually everywhere.*

From https://jmswen.wordpress.com/2011/06/05/a-probabilistic-proof-of-the-erdos-ko-rado-theorem/, Jon
Swenson.

> *Entering the keywords Erdős-Ko-Rado into MathSciNet gives a list of* **80 articles** *presenting Erdős-Ko-Rado results, but we should also count the original article of Erdős et al., which is not included since its title does not contain the words Erdős-Ko-Rado.*

From "Theorems of Erdős-Ko-Rado type in geometrical settings", Maarten de Boeck, Leo Storme, 2013.

Last week the count was **1,294 articles**, with 500 papers in the last 5 years.
There are several surveys [DF, E, FT, GM].

EKR for other obejcts

Object	Definition of intersection
k-Sets	a common element
Blocks in a design	a common element
k-Multi-sets	a common element
Vector spaces over a field	a common 1-D subspace
Lines in a partial geometry	a common point
Different geometries	common subspaces [DBS]
Integer sequences	same entry in same position
Permutations	both map i to j
Permutations in a Group	both map i to j
Permutations	a common cycle
Set Partitions	a common class
Tilings	a tile in the same place
Cocliques in a graph	a common vertex
Triangulations of a polygon	a common triangle

What is the size and structure of the largest set of intersecting objects?

General Framework

- Each **object** is made of k **atoms**.
- Two objects **intersect** if they contain a common atom.

Object	**Atoms**
Sets	elements from $\{1, \ldots, n\}$
Blocks of a design	elements from $\{1, \ldots, n\}$
multisets	elements from $\{1, \ldots, n\}$
Lines in a partial geometry	points in geometry
Integer sequences	pairs (i, a) (entry a is in position i)
Permutations	pairs (i, j) (the permutation maps i to j)
Permutations	cycles
Set partitions	subsets (cells in the partition)
Tilings	placement of tiles
Cocliques in a graph	vertices
Triangulations of a polygon	edge in the Triangulations
Triangulations of a polygon	Triangles in the Triangulations

- A **canonically intersecting family** is a set of all objects that contain a fixed atom.

Objects are said to have the **EKR property** if a canonically intersecting family is a largest intersecting family.

Variations and Related Results

1. **Characterize** all the intersecting families of maximum size.
2. For objects that have the EKR property, what is the largest intersecting family that is **not contained in a canonical** intersecting family?
   These are called Hilton-Milner Theorems or stability theorems
3. What is the size of the largest family so that any two sets have size **exactly** t.
4. r**-wise intersection**, so what is the largest family in which r of the objects have non-empty intersection.
5. Two families of objects from $[n]$, $\mathcal{A}$ and $\mathcal{B}$ arehang **cross-intersecting** if for every $A \in \mathcal{A}$ and every $B \in \mathcal{B}$ $A \cap B \neq \emptyset$. What are the largest cross-intersecting families?
   Results on cross-intersecting families can be used to get results on intersecting families-take $\mathcal{A} = \mathcal{B}$.

These questions all included in Erdős-Ko-Rado Combinatorics

Derangement Graphs

Definition

For a type of object, the **derangement graph for the object** has

- the vertices are the objects, and
- two vertices are adjacent if they are **not** intersecting.

- A **coclique/independent set/stable set** is a set of vertices in which no two are adjacent. The size of the largest coclique in X is denoted by $\alpha(X)$.
- A **clique** is a set of vertices in which every two are adjacent. The size of the largest clique in X is denoted by $\omega(X)$.

What is the size of the maximum coclique in a derangement graph?

Which cocliques achieve this bound?

The objects have the EKR property if the canonical cocliques are maximum cocliques.

Derangement Graphs

These are often well-known graphs:

Object	Derangement graph
Sets	Kneser graph
Blocks in a Design	Block Graph (SRG)
Vector spaces	q-Kneser graph
Integer sequences	n-Hamming graph
Permutations	Derangement graph Orignial Graphs
Triangulations of a polygon	$n-3$ distance graph of the associahedron

Derangement Graph for Sets—Kneser Graph

Definition

Define the Kneser graph $K(n, k)$

1. vertices are k-subsets of $\{1, \dots, n\}$; and
2. two k-sets are adjacent if they are disjoint.

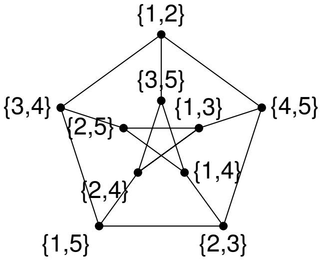


Figure: The Kneser Graph $K(5, 2)$, or our old friend Petersen. $\alpha(K(5,2)) = \binom{5-1}{2-1} = 4$

Kneser graphs are **vertex-transitive**, we can label the elements $\{1, 2, 3, 4, 5\}$ without changing the intersection.

Clique-Coclique Bound

A graph is **vertex-transitive** if the automorphism group of the graph is transitive on the vertices—all vertices are the same.

Lemma (Clique-coclique bound)

For a vertex-transitive graph X

$$\omega(X)\,\alpha(X) \leq |V(X)|.$$

Lots of the derangement graphs are vertex transitive.

Example

If $k|n$ then the maximum clique in $K(n,k)$ has size n/k

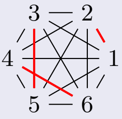
$$\{1,2,\dots,k\},\ \{k+1,k+2,\dots,2k\},\ \dots,\ \{n-k+1,n-k+2,\dots,n\}$$

So a coclique is no larger than $\frac{\binom{n}{k}}{n/k} = \binom{n-1}{k-1}$.

Perfect Matchings

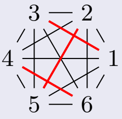
A **perfect matching** of $K_{2\ell}$ is a set of ℓ disjoint edges:

1 2 | 3 4 | 5 6



1 2 | 3 5 | 4 6

1 3 | 2 4 | 5 6



1 3 | 2 5 | 4 6

Two perfect matchings are intersecting if they have a common edge.

Derangement graph for perfect matchings on $K_{2\ell}$ has

- Vertex set all perfect matchings on $K_{2\ell}$ has size

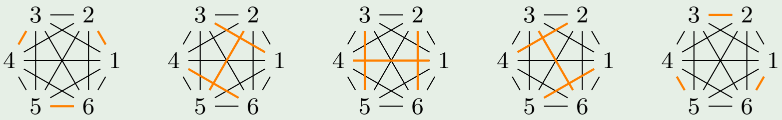
$$(2\ell - 1)!! = (2\ell - 1)(2\ell - 3)(2\ell - 5) \cdots 3 \cdot 1$$

- Two perfect matchings are adjacent if they don't have any common edges
- A coclique of all perfect matchings with a fixed edge has size $(2\ell - 3)!!$.
  This is a canonical intersecting family.
- This graph is vertex-transitive.
  Relabel the vertices of $K_{2\ell}$.

Design Theory

Example

A 1-factorization is a partition of the edges in the complete graph into perfect matchings.



A clique of size $2\ell - 1$ in the derangement graph is a 1-factorization

Theorem

The perfect matchings have the EKR-property.

Proof.

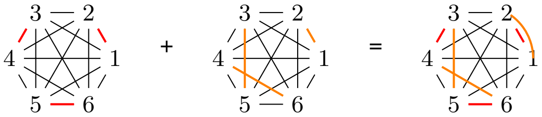
Any one-factorization is a clique with size $2\ell - 1$.
By the clique-clique bound a max set of intersecting perfect matchings has size

$$\frac{(2\ell-1)!!}{2\ell-1} = (2\ell-3)!!$$

which is the size of a canonical intersecting set.

Refinement of Derangement Graphs

The union of any two perfect matchings is a union of even cycles.



Pair of perfect matchings that are $[4, 2]$-related.

For $\lambda = (2\lambda_1, \ldots, 2\lambda_m)$ an even partition of 2ℓ define a graph X_λ

1. Vertex set is the set of all perfect matchings on $K_{2\ell}$.
2. Two perfect matchings are adjacent if their union is of type λ.
3. The derangement graph is the sum of all X_λ where λ doesn't have a part of size 2.

Example

Two perfect matchings are adjacent in the graph $X_{[2\ell]}$ if their union forms a Hamiltonian cycle. A clique of size $2\ell - 1$ in $X_{[2\ell]}$ is a **perfect** 1-factorization. These are conjectured to exist for any ℓ, but this is very open.

Association Scheme for Perfect Matchings

Definition

For each graph X_λ define a 01-matrix A_λ:

- rows and columns are indexed by the vertices of X_λ,
- the (u, v)-entry is 1, if the vertices are adjacent and 0 otherwise.

1. $A_\lambda = I$ if $\lambda = [2, 2, \ldots, 2]$ and $\sum A_\lambda$ is the all ones matrix,
2. Each A_λ is symmetric.
3. For any even partitions μ and ν

$$A_\mu \, A_\nu = \sum_\lambda p^\lambda_{\mu,\nu} A_\lambda$$

4. The algebra generated by all the A_λ is commutative and spanned by the $\{A_\lambda : \lambda \text{ an even partition of } 2\ell\}$.

Matrices with these properties are called an **Association Scheme**.

Eigenvalues for the Association Scheme on Perfect Matching

- The matrices $\{A_\lambda : \lambda \text{ an even partition of } 2\ell\}$ are simultaneously diagonalizable.
- There has been lots work to find nice equations for the eigenvalues of A_λ. [S]
  There is a formula that uses the irreducible characters of $\mathrm{Sym}(2\ell)$.
- There are interesting patterns in the eigenvalues [R, ZD]

Let Λ be the set of all even partitions of 2ℓ with no parts of size 2.

1. the adjacency matrix of the derangement graph is $A = \sum_{\lambda \in \Lambda} A_\lambda$.
2. A **weighted adjacency matrix** of the derangement graph is
$$A = \sum_{\lambda \in \Lambda} w_\lambda A_\lambda.$$
3. If ξ_λ^i is the eigenvalue of A_λ for the i^{th} common eigenspace, then the i^{th} eigenvalue of A is
$$\xi^i = \sum_{\lambda \in \Lambda} w_\lambda \xi_\lambda^i.$$

Why do we care about these eigenvalues?

Hoffman's bound/Delsarte's bound/Ratio bound

Ratio Bound

If A is a **weighted adjacency** matrix for a graph X then

$$\alpha(X) \leq \frac{|V(X)|}{1 - \frac{d}{\tau}}$$

where τ is the least eigenvalue and d is the largest eigenvalue of A.

See [H] for a nice history of this result.

Theorem

The perfect matchings have the EKR property

Proof.

The largest and smallest eigenvalues for the derangement graph are d_ℓ and $-\frac{d_\ell}{2\ell-2}$.
By the ratio bound

$$\alpha \leq \frac{(2\ell-1)!!}{1 - \frac{d_\ell}{-\frac{d_\ell}{2\ell-2}}} = (2\ell-3)!!$$

This works for different values of t for sets and perfect matchings [S, W]

Intersecting Permutations

Definition

Let G be a transitive permutation group, then two permutations $\sigma, \pi \in G$ **intersect** if for some $i \in \{1, \ldots, n\}$.

$$\sigma(i) = \pi(i) \quad \text{or} \quad \pi^{-1}\sigma(i) = i.$$

Permutations σ and π are intersecting if and only if $\pi^{-1}\sigma$ is **not** a derangement.

Example

1 → 2   5 → 4   1 → 2   5 → 4

$\sigma = (1, 2, 3)(4, 5)$ and $\pi = (1, 2)(3, 5, 4, 6)$ intersect since

$$\pi^{-1}\sigma = (1, 2)(3, 6, 4, 5)\ (1, 2, 3)(4, 5) = (1)(5)(2, 6, 4, 3) \qquad \leftarrow \text{two fixed points}$$

If G is transitive with degree n, the **canonical intersecting sets** are

$$S_{i,j} = \{\sigma \in G \mid i^{\sigma} = j\} \qquad \text{and} \qquad |S_{i,j}| = \frac{1}{n}|G|.$$

These are the cosets of a stabilizer of a point; call the **canonical intersecting set**.

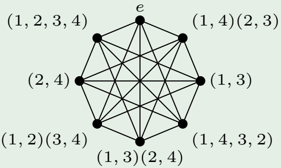
> For a transitive permutation group G what is the size of the largest set of intersecting permutations?

Derangement Graph for Groups

The **derangement graph** of G, denoted Γ_G has

1. the elements of G as its vertices, and
2. σ and π are adjacent if and only if $\pi^{-1}\sigma$ is **not** a derangement.

Example (Derangement graph for the Dihedral Group on 4 points)



A group has the **EKR property** if the canonical intersecting sets are the largest intersecting sets.

Depends on the group actions — as we learned on Wednesday: "Groups are meant to act".

Connections to Group Theory

- The derangement graph is connected if and only if the group is generated by its derangements.
  Almost all other groups, but this is an open problem [BCGR]
- Γ_G is the union of cliques if and only if G is a Frobenius group.
- If the group has a sharply transitive / regular subgroup (or subset), then the derangement graph has a clique the size of the degree
  By the clique/coclique bound this implies the group has the EKR property.[DF, CGS]

Normal Cayley Graphs

Definition

Let G be a group and C a subset of G. Define the **Cayley Graph** $Cay(G, C)$ to be the graph with

- the vertices elements of G,
- and g, h are adjacent if gh^{-1} is in the set C.

- $\Gamma_G = Cay(G, Der(G))$, the connection set is $Der(G)$, the set all of derangements of G.
- $Cay(G, C)$ is a **normal** Cayley graph if C is closed under conjugation.
- $\Gamma_G = Cay(G, Der(G))$ is a **normal** Cayley graph since $Der(G)$ is a union of conjugacy classes.

Theorem

The eigenvalues of $Cay(G, Der(G))$ *are*

$$\xi_\chi = \frac{1}{\chi(1)} \sum_{\sigma \in Der(G)} \chi(\sigma)$$

take over all χ *an irreducible character of* G*.*

Refinement the Derangement Graph.

For a conjugacy class C_i of G define a graph X_i by

- the vertices elements of G,
- and g, h are adjacent if gh^{-1} is in C_i.

Then

$$\Gamma_G = \bigcup_{C_i \text{ derangement}} X_i$$

The adjacency matrices $A_i = A(X_i)$ form an **Association Scheme**!

The adjacency matrix of Γ_G is

$$A_G = \sum_{C_i \text{ derangement}} A_i$$

Weightings for Derangement Graphs

1. The eigenvalues of X_i are
$$\xi_\chi = \frac{\chi(c_i)|C_i|}{\chi(\mathrm{id})}$$
where χ is an irreducible character of G and c_i is an element in C_i.
The eigenvalues of Γ_G can be very easy to calculate.
2. We can a weighted adjacency matrix by $A = \sum_{C_i \text{ derangement}} w_i A_i$, and the eigenvalues are
$$\xi_\chi = w_i \frac{\chi(c_i)|C_i|}{\chi(\mathrm{id})}.$$
3. For many groups it is not difficult to find a weighted adjacency matrix to optimize the ratio bound.
If we weight the entire conjugacy classes, this is an easy linear optimization problem.

Theorem (M, Spiga, Tiep)

All two transitive groups have the EKR property.

use a graph homomorphism, classification of finite simple groups and weighting.

Intersection Density

The **intersection density of** G is the ratio of the size largest intersecting set to the size of a stabilizer of a point. [LSP]

- The intersection density is always greater than or equal to 1.
- If the intersection density is 1, then the group has the EKR property.

What densities are possible and what group properties affect intersection density?

If the degree of the group is:

1. a prime power, the intersection density is one. [MRS]
2. $2p$ where p prime, the intersection density is either 1 or 2. [HKMM]

Link to data base of intersection density of small groups.

Using the Ratio Bound to get a Characterization*

Ratio Bound - second part

If equality holds in the ratio bound and S is a maximum coclique, then

$$v = v_S - \frac{|S|}{|V(X)|}\mathbf{1}$$

is a τ-eigenvector.

The ratio bound holds with equality for: sets, vector spaces, permutations, many permutations groups, perfect matchings, designs.

If equality holds in the ratio bound, define V be to the span of the characteristic vectors of all the canonical cocliques.

If this space is the span of the τ-eigenspace and the all ones vector:

1. The characteristic vector for any maximum intersecting set is in V.
2. Any maximum intersecting set will correspond to a 01-vector in V,
3. the number of 1s is the size of a maximum intersecting set.

Connect to Fourier Analysis*

For permutations this is Fourier Analysis on the symmetric group,

1. Consider the span of all functions with a Fourier transform is concentrated on specific irreducible representations.
2. This space of is spanned by characteristic functions for the canonical intersecting sets
3. Take the Fourier transform of the characteristic function for any coclique.
4. If the coclique is large, the Fourier transform is concentrated on specific irreducible representations.

Every analysis problem reduces to an easy combinatorics problem. Perhaps every combinatorics problem also reduces to an easy analysis problem?

> What are the 01-vectors in this vector space?

For many objects, these are considered to be **low dimensional Boolean Functions** [FKN, FI] or Cameron-Liebler sets [BSS, DMP], Perfect matchings [FL, S]

Probability—Robustness of the EKR theorem **

If Γ is the derangement graph for some object that has the EKR property, then we know the size of the largest coclique in the derangement graph.

At what probabilty can we randomly remove, without creating largest cocliques with high probability?

- For the k-sets if the probability is less than $\frac{\ln(n\binom{n-1}{k})}{\binom{n-k-1}{k-1}}$ then the cocliques don't get any bigger with high probability. [BBN, BKL, TD]
  this is the probability of forming a coclique by adding a single vertex to a maximum coclique.
- For permutations the analogous result holds.
- For perfect matchings he analogous result holds.

For a permutation group G the derangement graph is

$$\Gamma_G = Cay(G, \mathrm{Der}(G))$$

What happens if we make a new Cayley graph by randomly remove vertices from the connection set $\mathrm{Der}(G)$?

EKR combinatorics is not a rather dry topic, but rich and connected to many areas of math.

Thanks!

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