# Erdős-Ko-Rado Combinatorics

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## Definition

An **intersecting $k$-set family on an $n$-set** is

1. a collection of subsets from $\{1, 2, \ldots, n\}$,
2. each of size $k$, with
3. any two subsets from the family have at least one element in common.

A family is **$t$-intersecting** if any two sets from the system have at least $t$ elements in common.

## Example (A intersecting (7,3)-family)

$$\{\mathbf{1},\mathbf{2},\mathbf{3}\}\ , \{\mathbf{1},\mathbf{2},4\}, \{\mathbf{1},\mathbf{2},5\}, \{\mathbf{1},\mathbf{2},6\}, \{\mathbf{1},\mathbf{2},7\},$$
$$\{\mathbf{1},\mathbf{3},4\}, \{\mathbf{1},\mathbf{3},5\}, \{\mathbf{1},\mathbf{3},6\}, \{\mathbf{1},\mathbf{3},7\}, \{\mathbf{2},\mathbf{3},4\},$$
$$\{\mathbf{2},\mathbf{3},5\}, \{\mathbf{2},\mathbf{3},6\}, \{\mathbf{2},\mathbf{3},7\}$$

Every set has at least 2 elements from $\{1, 2, 3\}$ with size $\binom{n-3}{k-3} + \binom{3}{2}\binom{n-3}{k-2}$.

**Example (Another intersecting (7,3)-family)**

$$\{\mathbf{1},2,3\}, \{\mathbf{1},2,4\}, \{\mathbf{1},2,5\}, \{\mathbf{1},2,6\}, \{\mathbf{1},2,7\},$$
$$\{\mathbf{1},3,4\}, \{\mathbf{1},3,5\}, \{\mathbf{1},3,6\}, \{\mathbf{1},3,7\}, \{\mathbf{1},4,5\},$$
$$\{\mathbf{1},4,6\}, \{\mathbf{1},4,7\}, \{\mathbf{1},5,6\}, \{\mathbf{1},5,7\}, \{\mathbf{1},6,7\}$$

Every set contains 1 size $\binom{n-1}{k-1}$.

The family of all sets that contains a common element has size $\binom{n-1}{k-1}$; is called **canonically** or **trivially intersecting** or a **star** or **point pencil**.

The family of all sets that contains a common $t$-subset has size $\binom{n-t}{k-t}$; is called **canonically $t$-intersecting**.

# Erdős-Ko-Rado Theorem

## Theorem (Erdős-Ko-Rado, EKR )

*Let $\mathcal{F}$ be a $t$-intersecting $k$-set system on an $n$-set. If $n > f(k,t)$, then*

1. $|\mathcal{F}| \leq \binom{n-t}{k-t}$,
2. *and $\mathcal{F}$ meets this bound if and only if $\mathcal{F}$ is canonically $t$-intersecting.*

- For $t = 1$ this bound is $f(k,1) = 2k$.
- 1961 - Erdős, Ko and Rado had $f(k,t) \geq t + (k-t)\binom{k}{t}^3$ (but knew it wasn't exact).
- 1978 - Frankl proved $f(k,t) = (t+1)(k-t+1)$ when $t$ is large [F].
- 1984 - Wilson gave an algebraic proof with $f(k,t) = (t+1)(k-t+1)$ for all $t$ [W].
- 1997 - Ahslwede and Khachatrian found the largest $t$-intersecting family every triple $t, k, n$ [AK].

Last week a search on MathSciNet revealed **1,294 articles** Erdős-Ko-Rado results. There are several surveys [DF, E, FT, GM].

# EKR for Other Objects

| Object | Definition of intersection |
|---|---|
| $k$-Sets | a common element |
| Blocks in a design | a common element |
| Multi-sets | a common element |
| Vector spaces over a field | a common 1-D subspace |
| Lines in a partial geometry | a common point |
| Different geometries | common subspaces |
| Integer sequences | same entry in same position |
| Permutations | both map $i$ to $j$ |
| Permutations | a common cycle |
| Permutations in a group | both map $i$ to $j$ |
| Set Partitions | a common class |
| Trees on $n$ vertices | a common edge |
| Cocliques in a graph | a common vertex |
| Triangulations of a polygon | a common triangle |

For any object with a type of intersection, what is the largest intersecting family?

# General Framework

- Each **object** is made of $k$ **atoms**.

| Object | Atoms |
|---|---|
| Sets | elements from $\{1, \dots, n\}$ |
| Blocks of a design | elements from $\{1, \dots, n\}$ |
| multisets | elements from $\{1, \dots, n\}$ |
| Lines in a partial geometry | points in geometry |
| Integer sequences | pairs $(i, a)$ (entry $a$ is in position $i$) |
| Permutations | pairs $(i, j)$ (the permutation maps $i$ to $j$) |
| Permutations | cycles |
| Set partitions | subsets (cells in the partition) |
| Trees | $n - 1$ edges |
| Cocliques in a graph | vertices |
| Triangulations of a polygon | Triangles in the Triangulations |

- Two objects **intersect** if they contain a common atom.

**Canonically intersecting family** is a collection of all objects that contain a fixed atom.

# EKR-type Questions

1. What is the **size** of the largest intersecting families of objects?
2. **Characterize** all the intersecting families of largest size.

**EKR property**: a canonically intersecting family is a largest intersecting family.
**Strict EKR property**: only the canonically intersecting families are the largest intersecting families.

Related Intersection questions:

1. What is the size of the largest family so that any two sets have size **exactly** $t$.
2. $r$**-wise intersection**, so what is the largest family in which $r$ of the objects have non-empty intersection.
3. Two families of objects from $[n]$, $\mathcal{A}$ and $\mathcal{B}$ are **cross-intersecting** if for every $A \in \mathcal{A}$ and every $B \in \mathcal{B}$ $A \cap B \neq \emptyset$. What are the largest cross-intersecting families?
Results on cross-intersecting families can be used to get results on intersecting families—take $\mathcal{A} = \mathcal{B}$.

# Intersection Density

The **intersection density** for an object is the ratio of the size of the largest intersecting family to the size of the canonical intersecting family. [LSP]

- Intersection density is alway at least 1;
- Intersection density equals one if and only if the objects have the EKR property.

Intersection density measures how far objects are from having EKR property.

Related questions:

- For an object are there bounds on the intersection density?
- For an object what are the possible values of the intersecting density. Always rational, when it is integer?

# Stability and Robustness

- For objects that have the EKR property, what is the largest intersecting family that is **not contained in a canonical** intersecting family?
  These are called Hilton-Milner Theorems or stability theorems [HM]

  *"Stability versions of such theorems assert that if the size of a family is close to the maximum possible size, then the family itself must be close (in some appropriate sense) to a maximum-sized family."*

  –*Stability versions of Erdős-Ko-Rado type theorems, via isoperimetry* by Ellis, Keller, and Lifshitz

- What is the maximum family, if we relax the conditions on intersection?
  - Randomly allow some non-intersecting pairs of sets with a given probability.
  - **Almost intersecting** for each element in the family there is at most one other element that it doesn't intersect.

These result measure how **strong** the EKR property is for some objects.

# Proof Methods

1. The original proof used a method called **shifting** or **compression**. This is an operation on subsets that takes the entire family and shifts it to an intersecting family in which all sets are smaller in co-lex ordering.
   Shifting can be a tricky operation to use!!
2. A simple counting argument, called the **kernel method** can work asymptotically.
3. There is a nice proof by Katona that uses a subfamily.
   Wikipedia has a version of this proof that is interesting and accessible at the high school level.
4. My preferred method represent the problem in a **graph** and use algebraic graph theory.

For a set of objects, define the **derangement graph** of the object

- the vertices are the objects,
- two vertices are adjacent if they are **not** intersecting.

- A coclique in the derangement graph is an intersecting family of objects.
  A **coclique/independent set** is a set of vertices in which no two are adjacent.

What is the size of the maximum coclique in a derangement graph?

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Which cocliques achieve this bound?

- The objects have the EKR property if the canonical intersecting families are maximum cocliques (call these **canonical cocliques**).
- Some of the related questions also can be phrased as a graph question.

# Examples of Derangement Graphs

| Object | Derangement graph |
|---|---|
| Sets | Kneser graph |
| Blocks in a Design | Block Graph (SRG) |
| Vector spaces | $q$-Kneser graph |
| Integer sequences | $n$-Hamming graph |
| Permutations | Derangement graph |
| Triangulations of a polygon | $n-3$ distance graph of the associahedron |

Define the Kneser graph $K(n,k)$

1. vertices are $k$-subsets of $\{1,\ldots,n\}$;
2. two $k$-sets are adjacent if they are disjoint.

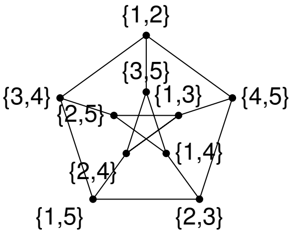



Figure: The Kneser Graph $K(5,2)$, or our old friend Petersen.

## Definition

Let $X$ and $Y$ be graphs. A **graph homomorphism** $f$ of $X$ to $Y$ is a map $f : V(X) \to V(Y)$ such that if $v \sim_X w$, then $f(v) \sim_Y f(w)$.

## Lemma (No-Homomorphism Lemma)

*If $X$ and $Y$ are vertex-transitive graphs and $X \to Y$, then*

$$\alpha(Y) \leq \frac{|V(Y)|}{|V(X)|}\alpha(X)$$

Proof.

- Fractional chromatic number satisfies $\chi^*(X) \leq \chi^*(Y)$ since $X \to Y$,
- Since $X$ and $Y$ are vertex transitive, their **fractional chromatic numbers** are

$$\chi^*(X) = \frac{|V(X)|}{\alpha(X)} \quad \text{and} \quad \chi^*(Y) = \frac{|V(Y)|}{\alpha(Y)}.$$

Define $C(n,k)$ to be graph with vertices cyclic $k$-intervals from $\{1,\ldots,n\}$ and two intervals are adjacent if they are disjoint.

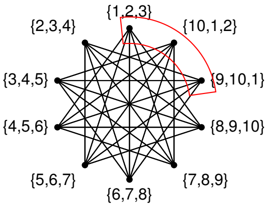



Figure: The graph $C(10,3)$.

- $C(n,k)$ is a subgraph of $K(n,k)$.
- $C(n,k)$ is vertex transitive.
- $\alpha(C(n,k)) = k$.
- $\chi^*(C(n,k)) = \frac{n}{k}$.

There is a homomorphism $C(n,k) \to K(n,k)$, by the no homomorphism lemma:

$$\alpha(K(n,k)) \leq \frac{\binom{n}{k}}{n} k = \binom{n-1}{k-1}.$$

A **weighted adjacency matrix** of a graph is a

- symmetric matrix with rows/columns indexed by the vertices of the graph;
- and the $(v, w)$-entry is 0 if $v$ and $w$ are not adjacent.

**Theorem**

*Let $A$ a weighted adjacency matrix for the graph $X$ with constant row sum $d$. Then*

$$\alpha(X) \leq \frac{|V(X)|}{1 - \frac{d}{\tau}}$$

*where $\tau$ is the least eigenvalue of $A(X)$. If equality holds and $S$ is maximum coclique,*

$$v_S - \frac{\alpha(X)}{|V(X)|}\mathbf{1}$$

*is an eigenvector for $\tau$ where $v_S$ is the characteristic vector of $S$.*

Willem H. Haemers wrote a nice paper on this bound [H]

# Wilson's Proof for sets

Define $A(n,k,i)$ adjacency matrix for $k$-sets with entry equal to 1 if $|A \cap B| = k-i$

1. $A = \sum_{i=k-t+1}^{k} A(n,k,i)$ is the adjacency matrix for the derangement graph for $t$-intersecting sets.
   Rows and columns correspond to $k$-sets, two are adjacent if they are not $t$-intersecting.
2. The $A(n,k,i)$ commute so they are simultaneously diagonalizable and the eigenvalues are known.
3. Wilson gave a weighted adjacent matrix for this graph

$$A = \sum_{i=k-t+1}^{k} w_i A(n,k,i)$$

   are: $\binom{n}{k}\binom{n-t}{k-t}^{-1} - 1$ , or $-1$ , or larger than $-1$.
4. This matrix gives equality in ratio bound.

We can get the characterization of the maximum intersecting sets from the eigenspace of the least eigenvalue.

# Permutations

## Definition

Let $G$ be a transitive permutation group, then two permutations $\sigma, \pi \in G$ **intersect** if for some $i \in \{1, \dots, n\}$.

$$\sigma(i) = \pi(i) \quad \text{or} \quad \pi^{-1}\sigma(i) = i.$$

Permutations $\sigma$ and $\pi$ are intersecting if and only if $\pi^{-1}\sigma$ is **not** a derangement.

## Example

$\sigma = (\overset{1 \to 2}{1, 2}, 3)(\overset{5 \to 4}{4, 5})$ and $\pi = (\overset{1 \to 2}{1, 2})(3, \overset{5 \to 4}{5, 4}, 6)$ intersect since

$$\pi^{-1}\sigma = (1,2)(3,6,4,5)\ (1,2,3)(4,5) = (1)(5)(2,6,4,3) \quad \leftarrow \text{two fixed points}$$

## Example

$\sigma = (1,4,3,2)(5,6)$ and $\rho = (1,2,3)(4,5)$ don't intersect since

$$\rho^{-1}\sigma = (1,3,2)(4,5)(1,4,3,2)(5,6) = (1,5,6,4,2,3) \quad \leftarrow \text{no fixed points}$$

# Permutations

## Definition

For a transitive group $G$ with degree $n$, the set

$$S_{i,j} = \{\sigma \in G \,|\, i^\sigma = j\}.$$

is a **canonical intersecting set** with size $\frac{|G|}{n}$.

A canonical intersecting is the coset of a stabilizer of a point

A permutation group $G$ has the EKR property if a stabilizer of a point is a largest intersecting set.

Depends on the group action!

For a $t$-transitive group $G$, the coset of stabilizer of a $t$-set is a **canonical $t$- intersecting set** with size $\frac{|G|}{n(n-1)\dots(n-t+1)}$.

**Definition**

For any $G \leq \mathrm{Sym}(n)$ we can define the **derangement graph**, $\Gamma_G$.

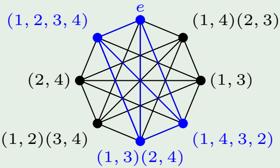
- The vertices are the elements of $G$.
- Vertices $\sigma, \pi \in G$ are adjacent if and only if $\pi^{-1}\sigma$ is a derangement.

Permutations are adjacent if they are **not** intersecting.

- An intersecting set in $G$ is a **coclique** (independent set) in $\Gamma_G$.
- The **degree** of any vertex equal the number of derangements in $G$.
- The derangement graph is **vertex transitive**, $G$ acts transitively on the vertices $\Gamma_G$.
- $\Gamma_G$ is the union of cliques if and only if $G$ is a Frobenius group.
- The derangement graph is connected if and only if the group is generated by its derangements.
  Connected for lmost all non-Frobenius groups, but this is an open problem [BCGR]

# Example and Homomorphism

## Example




The graph $\Gamma_{D(4)}$.

There is a homomorphism $K_4 \to \Gamma_{D(4)}$, from the subgroup $C_4$, so

$$\alpha(\Gamma_{D(4)}) \leq \frac{8}{4}\,\alpha(K_4) = \frac{|D(4)|}{4} = 2.$$

This is also the clique-coclique bound.

## Lemma

*If $G$ contains a regular subgroup, then $G$ has the EKR property.*

# Eigenvalues of Cayley Graphs

1. The derangement graph is a **Cayley graph**

$$\Gamma_G = \mathrm{Cay}(G, \mathrm{Der}(G))$$

   where the connection set is $\mathrm{Der}(G)$, the set of derangements of $G$.
   The vertices are elements of $G$ and $\sigma\pi$ are adjacent if $\pi^{-1}\sigma \in \mathrm{Der}(G)$.
2. The derangement graph is a **normal** Cayley graph, connection set is closed under conjugation.
3. The connection set $\mathrm{Der}(G)$ is the union of conjugacy classes, so it is closed under conjugation.

### Theorem

*If $\mathrm{Cay}(G, C)$ is a* **normal** *Cayley graph, then its eigenvalues are*

$$\frac{1}{\chi(1)} \sum_{\sigma \in C} \chi(\sigma)$$

*where $\chi$ is an irreducible character of $G$.*

The eigenvalues of the derangement graph are not so difficult to calculate.

The eigenvalues of the derangement graph can tell us information about its structure.

- The graph is connected if the degree has multiplicity 1.
- If the least eigenvalue is $-1$, then the graph is the union of complete graphs
- If difference of the degree and the least eigenvalue is the number of vertices, then the graph is a **join**.

## Definition

A **cograph**, or complement-reducible graph, is a graph that can be generated from the single-vertex graph $K_1$ by complementation and disjoint union.

It is easy to spot a vertex-transitive cograph from its eigenvalues, and many of the derangement graphs are co-graphs.

I don't know why or when a derangement graph is a cograph!

# Characterization for Permutations

**Theorem**

*The symmetric group* $\mathrm{Sym}(n)$ *has the strict EKR property.*

Any intersecting family of all permutations of maximum size is the coset of a stabilizer of a point.

Brief history of the proof

1. Bound by Deza and Frankl [DF] (1977), they conjectured the characterization.
2. Cameron and Ku [CK] (2003) proved the characterization by a form of shifting (only works since there is a large clique)
3. Larose and Malvenuto [LM] (2003) gave an alternate proof.
4. Wang and Zhang [WZ] (2008) used clever repeated application of the clique/coclique bound.
5. Godsil and M. [GM] (2009) used the ratio bound
6. Ellis, Friedgut and Pilpel [EFP] (2011) proved the EKR theorem for $t$-intersecting permutations and $n$ large.
7. Chase, Dafni, Filmus and Lindzey [CDFL] (2022) gave a nice characterization for the maximum sets with larger $t$.

# Refinement the Derangement Graph.

For a conjugacy class $C_i$ of $G$ define a graph $X_i$ by

- the vertices elements of $G$,
- and $\sigma, \pi$ are adjacent if $\pi^{-1}\sigma \in C_i$.

Set $A_i = A(X_i)$, then

- $$\Gamma_G = \bigcup_{C_i \text{ derangement}} X_i, \quad \text{and} \quad A(\Gamma_G) = \sum_{C_i \text{ derangement}} A_i$$
- $A_0 = I$ if $C_0 = \{e\}$ and $\sum A_\lambda$ is the all ones matrix,
- Each $A_i$ is symmetric.
- For any $i$ and $j$
$$A_i \, A_j = \sum_{\lambda} p_{i,j}^k A_k$$
- The algebra generated by all the $A_i$ is commutative and spanned by the $\{A_i : C_i \text{ is a conjugacy class of } G\}$.

The adjacency matrices $A_i$ form an **Association Scheme**!

# Weightings for Derangement Graphs

1. The eigenvalues of $X_i$ are
$$\xi_\chi = \frac{\chi(c_i)|C_i|}{\chi(\mathrm{id})}$$
where $\chi$ is an irreducible character of $G$ and $c_i$ is an element in $C_i$.
The eigenvalues of $\Gamma_G$ can be very easy to calculate.
2. A weighted adjacency matrix for a derangement graph be formed like:
$$A = \sum_{C_i \text{ derangement}} w_i A_i,$$
3. and the eigenvalues of the weighted matrix are
$$\xi_\chi = \sum_{C_i \text{ derangement}} w_i \frac{\chi(c_i)|C_i|}{\chi(\mathrm{id})}.$$
4. For many groups it is not difficult to find $w_i$ so that the ratio bound is tight.

**Theorem (M, Spiga, Tiep)**

*All two transitive groups have the EKR property.*

Use a graph homomorphism, classification of finite simple groups and weighting.

# Intersection Density

The **intersection density of a transitive group** $G$ is the ratio of the size largest intersecting family, to the size of a stabilizer of a point. [LSP]

Always at least 1, equal to 1 exactly when the group has the EKR property.

If the **degree** of the group is :

1. $n > 2$, the intersection density is no more than $n/3$.
2. a prime power, the intersection density is one. [MRS]
3. $2p$ where $p$ prime, the intersection density is either 1 or 2. [HKMM]
4. is $pq$, two primes with $q < p$, the intersection density of many of the group is 1; there are examples where it is $q$.
   The construction depends heavily on certain equidistant cyclic codes over the field $\mathbb{F}_q$

Link to data base of intersection density of small groups.

# Robustness of the EKR theorem *

If $\Gamma$ is the derangement graph for some object that has the EKR property, can we randomly remove edges, without creating a larger coclique with high probability?

- For the $k$-sets if we remove edges from the Kneser graph $K(n,k)$ probability is less than $\frac{\ln(n\binom{n-1}{k})}{\binom{n-k-1}{k-1}}$ then the cocliques don't get any bigger with high probability. [BBN, BKL, TD]
  this is the probability of forming a coclique by adding a single vertex to a maximum coclique.
- For perfect matchings and permutation the analogous result holds [GMMPS].

For a permutation group $G$ the derangement graph is

$$\Gamma_G = Cay(G, \mathrm{Der}(G))$$

What happens if we make a new Cayley graph from a derangement graph by randomly remove vertices from the connection set $\mathrm{Der}(G)$? [GMMPS2]

# Using the Ratio Bound to get a Characterization **

## Ratio Bound - second part

If equality holds in the ratio bound and $S$ is a maximum coclique, then

$$v_S - \frac{|S|}{|V(X)|}\mathbf{1}$$

is a $\tau$-eigenvector ($\tau$ is the least eigenvalue).

The ratio bound holds with equality for: sets, vector spaces, many permutations groups, perfect matchings, designs.

Let $V$ be the span of the characteristic vectors of all the canonical cocliques.

If equality holds in the ratio bound, then $V$ is a **subspace** of the span of the $\tau$-eigenspace and the $d$-eigenspace. If these spaces are equal, we have a method to get the characterization:

1. The characteristic vector for any maximum intersecting set will be a linear combination of the characteristic vectors of the canonical cocliques.

# More on the characterization **

For permutations this approach is uses finite Fourier analysis on the group [FL].

1. Often $V$ is span of all functions with a Fourier transform that is concentrated on specific irreducible representations.
   Representations in the decomposition of the permutation representation.
2. Take the Fourier transform of the characteristic function for any coclique.
3. If the coclique is large, the Fourier transform is concentrated on specific irreducible representations.

We can also ask what are the 01-vectors in $V$?

For many objects, these are considered to be **low dimensional Boolean Functions** [FKN, FI] or **Cameron-Liebler sets** [BSS, DMP].

# Intersecting Trees ***

Two trees on the same vertex set are **intersecting** if they have a common edge.

What is the size of the largest family of intersecting trees on $n$ vertices?

1. A **canonical** intersecting family of trees is the set of all trees that contain a common edge, the size is $2n^{n-3}$.
2. Any star (tree formed by taking all edges on a fixed vertex) is intersecting with every other tree

Not all trees are the same!

**Theorem (FHIKLMP)**

*For $n \geq 2^{19}$, let $\mathcal{F}$ be a $1$-intersecting family of trees on $n$ vertices. Then*

$$|\mathcal{F}| \leq 2n^{n-3} + (n-2).$$

*Equality holds if and only if $\mathcal{F}$ consists of all trees containing a fixed edge and all stars.*

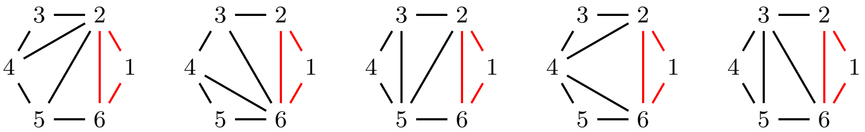
There is also a $t$-intersecting theorem.

# Outline of the Spread Approximation Technique [KZ] ***

1. First identify the objects as sets, denoted bt $\mathcal{A}$. Represent a tree by its set of edges.
   This is a $(n-1)$-subset from $\{1, 2, \ldots, \binom{n}{2}\}$
2. A type of object $\mathcal{A} \subset 2^{[m]}$ is $r$**-spread** if the number of elements in $\mathcal{A}$ that contain an $r$-subset $X$ is bounded above by $r^{-|X|}\,|\mathcal{A}|$.
   The **spreadedness** captures the amount of quasi-randomness of the objects
3. If $\mathcal{A}$ is spread and $\mathcal{F} \subseteq \mathcal{A}$ is a family of $t$-intersecting subsets no bigger than $k$
   1. A $t$-intersecting family of smaller subsets $\mathcal{S}$ called the **spread approximation** of $\mathcal{F}$.
   2. **Most** of $\mathcal{F}$ contain a set from $S$.
   3. An intersecting family in $\mathcal{A}$, all containing a set in $S$, is no bigger than a canonical family.
4. Take $\mathcal{F}$ into two piles: those contained in spread approximation, and the leftovers.
   1. The leftover pile is always small.
   2. If the spread approximation is not canonical, then the part contained in it is small.
   3. If the spread approximation is a canonical family, and the leftover pile is not empty then the family is small.
      The number of sets removed from the canonical family by an element in the leftover pile, is more than the elements in the left-overs pile.

# Triangulations of Polygons ☠

- Start with a convex $n$-gon.
- Make a triangulation by adding $n-3$ edges that only intersect at vertices of the $n$-gon.
- Intersect if they have a common triangle.




What is the largest family of intersecting triangulations?

- The number of triangulations is the $(n-2)^{th}$-Catalan number, and
- the number of triangulations with a short edges is the $(n-3)^{th}$-Catalan number
- See Gil Kalai's "Intersecting family of triangulations." MathOverflow. mathoverflow.net/q/114646.

## Thanks!

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