

Math 103-001 Winter 2015 Quiz #5

1. Evaluate the following indefinite integrals:

$$a) \int (\sqrt[5]{x} + \frac{2}{x^3} + 4e^x) dx \quad [8 \text{ marks}]$$

$$= \int (x^{1/5} + 2x^{-3} + 4e^x) dx$$

$$= \frac{5}{6} x^{6/5} - x^{-2} + 4e^x + C$$

$$b) \int \frac{(x+1)^2}{x^2} dx = \int \frac{x^2 + 2x + 1}{x^2} dx$$

$$= \int \left(1 + \frac{2}{x} + \frac{1}{x^2}\right) dx$$

$$= x + 2 \ln|x| - x^{-1} + C$$

$$c) \int_1^3 (3x - 9x^{-2}) dx = \left[\frac{3}{2} x^2 + 9x^{-1} \right]_1^3$$

$$= \left(\frac{3}{2} \cdot 9 + 3 \right) - \left(\frac{3}{2} + 9 \right)$$

$$= 6$$

2. The velocity function of an object is given by $v(t) = 3t^4 - 2t^3 + 3t^2$ Find the position function $s(t)$, given that at time $t=0$, the position is $s(0)=4$.

[4 marks]

$$s(t) = \int v(t) dt$$

$$= \frac{3}{5} t^5 - \frac{1}{2} t^4 + t^3 + C$$

$$\text{FIND } C: \quad s(0) = 4 \quad \text{so} \quad C = 4$$

$$\Rightarrow s(t) = \frac{3}{5} t^5 - \frac{1}{2} t^4 + t^3 + 4$$

3. Integrate by substitution:

[8 marks]

a) $\int x e^{-x^2} dx$

LET $u = -x^2$

$$\frac{du}{dx} = -2x$$

$$x dx = -\frac{1}{2} du$$

$$= -\frac{1}{2} \int e^u du$$

$$= -\frac{1}{2} e^u + C$$

$$= -\frac{1}{2} e^{-x^2} + C$$

b) $\int \frac{x+3}{(x^2+6x-2)^2} dx$

LET $u = x^2+6x-2$

$$\frac{du}{dx} = 2x+6$$

$$\frac{1}{2} du = (x+3) dx$$

$$= \frac{1}{2} \int \frac{du}{u^2}$$

$$= \frac{1}{2} \int u^{-2} du = -\frac{1}{2} u^{-1} + C$$

$$= -\frac{1}{2} (x^2+6x-2)^{-1} + C$$