Tuesday March 24th , 2015

Name: SOLUTIONS

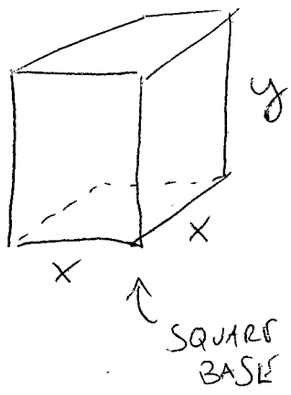
ID #:

Math 103-001 Winter 2015 Quiz #~~3~~4

1. A closed rectangular storage container with square base and a volume of 54 m^3 is to be constructed out of two different types of material.

The bottom is made of metal and costs \$300 per m^2, the remaining five sides are made of wood and cost \$100 per m^2. Find the optimal dimensions of such a container, i.e. the dimensions of a container that has minimal material cost. Show all steps clearly and give your final answer in a complete sentence.

[10 marks]



OBJECTIVE: MINIMIZE COST

C = TOP + 4 SIDES + BOTTOM

$= 100x^2 + 400xy + 300x^2$

$= 400x^2 + 400xy$

CONSTRAINT: VOLUME = 54 m^3

$x^2 y = 54$

$\Rightarrow$ COMBINE: $y = \frac{54}{x^2}$

SO COST $C = 400x^2 + 400 \times \left(\frac{54}{x^2}\right)$

$C(x) = 400x^2 + 21600\,x^{-1}$

NOW MINIMIZE $C(x)$:

$C'(x) = 800x - 21600\,x^{-2}$

$C''(x) = 800 + 43200\,x^{-3}$

FIND CRITICAL POINTS:

$C'(x) = 0 \Rightarrow 800x = 21600\,x^{-2}$

$x^3 = 27$

$x = 3$

$C'(x)$ DNE $\Rightarrow$ NO SUCH POINT.

TEST $C''(3) > 0$, SO $x = 3$ IS A LOCAL MINIMUM.

[ALSO NO ENDPOINTS, SO $x = 3$ IS THE GLOBAL MIN]

THE DIMENSIONS ARE $x = 3$ AND $y = \frac{54}{3^2} = 6$

IE $3m \times 3m \times 6m$

2. Consider an investment of $4000. Find the value of the investment in 20 years, if it grows at an annual interest rate of 2.5%, compounded *[4 marks]*

i) monthly $B(t) = P\left(1 + \frac{r}{k}\right)^{kt}$, $P = 4000$, $r = 0.025$, $k = 12$, $t = 20$

$$\Rightarrow B(20) = 4000\left(1 + \frac{.025}{12}\right)^{12 \cdot 20} = \$6591.46$$

ii) continuously

$$B(t) = Pe^{rt}$$

$$\Rightarrow B(20) = 4000\, e^{(0.025)(20)} = \$6594.88$$

3. An investment is earning compound interest continuously. If it doubles in value every 15 years, how long will it take to triple in value? Show all work. *[6 marks]*

USE $B(t) = Pe^{rt}$

IN $t = 15$ YEARS, $B(t) = 2 \cdot P$ (DOUBLED)

IE $2P = Pe^{r \cdot 15}$

SOLVE FOR $r = \frac{\ln(2)}{15}$ $\left(\approx .0462\right)$

NOW FIND TIME t FOR $B(t)$ TO TRIPLE:

$3P = Pe^{\frac{\ln 2}{15} t}$

$$\ln 3 = \frac{\ln 2}{15} \cdot t$$

$$t = \frac{15 \cdot \ln 3}{\ln 2} \approx 23.77$$

IT WILL TAKE 23.77 YEARS TO TRIPLE.