

Math 103-001 Winter 2015 Quiz #3

1.

[6 marks]

a) State the limit definition of the derivative function $f'(x)$.

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

b) Using the limit definition, evaluate the derivative of the function $f(x) = x - 2x^2$

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{(x+h) - 2(x+h)^2 - (x - 2x^2)}{h} \\ &= \lim_{h \rightarrow 0} \frac{x+h - 2x^2 - 4xh - 4h^2 - x + 2x^2}{h} \\ &= \lim_{h \rightarrow 0} \frac{h - 4xh - 4h^2}{h} \\ &= \lim_{h \rightarrow 0} (1 - 4x - 4h) \\ &= 1 - 4x \end{aligned}$$

2. The debt of a small country (in millions of \$) in t years is given by the function

$$d(t) = 3t^2 - 24t + 53$$

What is the debt, and at what rate is the debt increasing or decreasing (give units) in exactly 3.5 years? (You do not need to use the limit definition for this question)

[4 marks]

$$\begin{aligned} \text{DEBT: } d(3.5) &= 3(3.5)^2 - 24(3.5) + 53 \\ &= 5.75 \end{aligned}$$

$$\text{R.O.C.: } d'(t) = 6t - 24$$

$$d'(3.5) = 6(3.5) - 24 = -3$$

IN 3.5 YEARS, THE DEBT WILL BE \$5.75 MILLION
AND DECREASING AT A RATE OF \$3 MILLION/YEAR.

3. Use the derivative rules to find the derivative $f'(x)$ in each case.

You do not need to simplify your answers.

[6 marks]

a) $f(x) = 5x^4 + \sqrt[3]{x} - \frac{2}{x^5} \quad \left(= 5x^4 + x^{1/3} - 2x^{-5} \right)$

$$f'(x) = 20x^3 + \frac{1}{3}x^{-2/3} + 10x^{-6}$$

b) $f(x) = (3x^2 - 5x + 3)(x^6 + 7x^{-2} - 2x)$

$$f'(x) = (6x - 5)(x^6 + 7x^{-2} - 2x) + (3x^2 - 5x + 3)(6x^5 - 14x^{-3} - 2)$$

c) $f(x) = \frac{3}{x^4 + x^2 - 1}$

$$f'(x) = \frac{-3(4x^3 + 2x)}{(x^4 + x^2 - 1)^2}$$

4. If $g(t) = \frac{4}{5}t^{5/4} + \frac{18}{5}t^{-2/3}$, find and simplify $g''(1)$.

[4 marks]

$$g'(t) = t^{1/4} - \frac{12}{5}t^{-5/3}$$

$$g''(t) = \frac{1}{4}t^{-3/4} + 4t^{-8/3}$$

$$g''(1) = \frac{1}{4} + 4 = 4\frac{1}{4} \quad \left(\text{or } \frac{17}{4} \right)$$