Tuesday February 3rd, 2015

Name: SOLUTIONS

ID #:

Math 103-001 Winter 2015 Quiz #2

1. Evaluate each limit. If a limit does not exist, evaluate both one sided limits separately.

[10 marks]

a) $\lim_{x\to 2}\frac{x^2+x-6}{x^3-4x} = \lim_{x\to 2}\frac{(x+3)(x-2)}{x(x-2)(x+2)}$

"$\frac{0}{0}$" $= \lim_{x\to 2}\frac{x+3}{x(x+2)} = \frac{2+3}{2(2+2)} = \frac{5}{8}$

b) $\lim_{x\to 4}\frac{\sqrt{x}-2}{4-x} \cdot \frac{\sqrt{x}+2}{\sqrt{x}+2} = \lim_{x\to 4}\frac{x-4}{(4-x)(\sqrt{x}+2)}$

"$\frac{0}{0}$" $= \lim_{x\to 4}\frac{-1}{\sqrt{x}+2}$

$= \frac{-1}{4}$

c) $\lim_{x\to -1}\frac{x+1}{x^3+2x^2+x} = \lim_{x\to -1}\frac{x+1}{x(x+1)^2}$

"$\frac{0}{0}$" $= \lim_{x\to -1}\frac{1}{x(x+1)}$ "$\frac{1}{0}$" $\Rightarrow$ LIMIT D.N.E.

FROM LEFT: $\lim_{x\to -1^-}\frac{1}{\underset{\text{NEG}}{x}\,\underset{\text{NEG}}{(x+1)}} = +\infty$

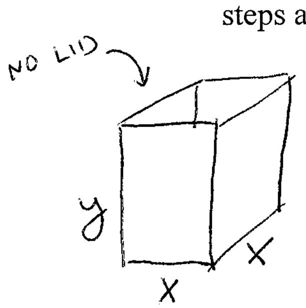
FROM RIGHT: $\lim_{x\to -1^+}\frac{1}{\underset{\text{NEG}}{x}\,\underset{\text{POS}}{(x+1)}} = -\infty$

2. Evaluate the limit $\lim_{x\to\infty} \frac{3x-4x^2}{x^2+2}$ DIVIDE BY x^2 *[3 marks]*

$$= \lim_{x\to\infty} \frac{3/x - 4}{1 + \frac{2}{x^2}} = \frac{0-4}{1+0} = -4$$

3. An open rectangular box has a square base and no lid. It must contain a volume of 14 m^3. We wish to find the amount of material required to build this box, i.e. its surface area. *[7 marks]*

a) Express the surface area A as a function of the box's width x, i.e. find A(x). Show all steps and include a sketch of the box.



VOLUME $V = \text{LENGTH} \cdot \text{WIDTH} \cdot \text{HEIGHT}$

$= x \cdot x \cdot y = x^2 y$

GIVEN: $x^2 y = 14$

SURFACE AREA $A = \text{BASE} + 4 \text{ SIDES}$

$= x^2 + 4(xy)$

ELIMINATE y BY SOLVING $x^2 y = 14$

FOR $y = \frac{14}{x^2}$

AND SUBSTITUTE: $A(x) = x^2 + 4 \times \left(\frac{14}{x^2}\right)$

$= x^2 + \frac{56}{x}$

b) Due to other considerations, you can either build a box that has width 2m, a box that has width 2.5m, or a box that has width 3m. Of these three options, which is the most economical design in terms of material used? (GIVE ~~ALL~~ DIMENSIONS)

TRY EACH WIDTH:

$x = 2 \rightarrow A(2) = 32 \ m^2$

$x = 2.5 \rightarrow A(2.5) = 28.65 \ m^2$

$x = 3 \rightarrow A(3) = 27\frac{2}{3} \ m^2$

SO THE OPTIMAL DIMENSIONS ARE $3 \times 3 \times \frac{14}{9}$ m