Tuesday March 10th

Name: ______________________

ID #: ______________________

Math 103-001 Winter 2015 Midterm

1. Consider the function $f(x) = \dfrac{2x-1}{x^2-9}$ *[5 marks]*

a) Find and state the domain of f(x).

CAN'T DIVIDE BY ZERO:

NEED $x^2 - 9 = 0$

$(x-3)(x+3) = 0 \implies$ DOMAIN IS ALL x EXCEPT $x = \pm 3$

b) Find and state all x- and y-intercepts of f(x).

Y-INT: $f(0) = 1/9$

X-INT: SOLVE $0 = \dfrac{2x-1}{x^2-9}$

$\implies 0 = 2x-1$

$\implies x = 1/2$

Y-INT: $y = 1/9$

X-INT: $x = 1/2$

2. The vendor at a local farmers market is selling home-made pies. From previous experience they know that they can sell 33 pies per weekend if the pies are priced at \$16 each. Reducing the price to \$10 each has resulted in sales of 51 pies per weekend. Find the demand function p(x), i.e. price p as a function of pies sold x. You may assume such a demand function to be linear. *[4 marks]*

LINEAR, IE $p(x) = mx + b$

SLOPE $m = \dfrac{\text{RISE}}{\text{RUN}} = \dfrac{16-10}{33-51} = -\dfrac{6}{18} = -\dfrac{1}{3}$

INTERCEPT: SOLVE $16 = \left(-\frac{1}{3}\right)(33) + b \implies b = 27$

THE DEMAND FUNCTION IS

$$p(x) = -\frac{1}{3}x + 27$$

3. Find the following limits. If a given limit does not exist, state so. *[9 marks]*

a) $\lim\limits_{x\to 3}\dfrac{9-x^2}{x^2+x-12}$ $= \lim\limits_{x\to 3} \dfrac{(3-x)(3+x)}{(x+4)(x-3)}$

$= -\dfrac{\cancel{8}\,6}{7}$

b) $\lim\limits_{x\to 3}\dfrac{\frac{1}{3}-\frac{1}{x}}{x-3}$ $= \lim\limits_{x\to 3} \dfrac{\frac{x-3}{3x}}{x-3}$

$= \lim\limits_{x\to 3} \dfrac{1}{3x} = \dfrac{1}{9}$

c) $\lim\limits_{x\to \infty}\dfrac{1+2x-3x^3}{6x^3+5x}$ $= \lim\limits_{x\to\infty} \dfrac{\frac{1}{x^3}+\frac{2}{x^2}-3}{6+\frac{5}{x^2}}$

$= -\dfrac{3}{6} = -\frac{1}{2}$

4. Given the function $f(x)=\sqrt{x-1}$, evaluate $\lim\limits_{x\to 5}\dfrac{f(x)-f(5)}{x-5}$ *[4 marks]*

$\lim\limits_{x\to 5} \dfrac{\sqrt{x-1}-\sqrt{5-1}}{x-5} = \lim\limits_{x\to 5} \dfrac{\sqrt{x-1}-2}{x-5}\cdot\dfrac{\sqrt{x-1}+2}{\sqrt{x-1}+2}$

$= \lim\limits_{x\to 5} \dfrac{(x-1)-(4)}{(x-5)(\sqrt{x-1}+2)}$

$= \lim\limits_{x\to 5} \dfrac{1}{\sqrt{x-1}+2} = \dfrac{1}{\sqrt{5-1}+2} = \dfrac{1}{4}$

5. Find the indicated derivatives using the rules of differentiation. You do not need to simplify your answers. *[12 marks]*

a) $f(x) = x^3 - \frac{3}{x} + 3x - \sqrt[3]{x}$ $\qquad f''(x) = ?$

$$= x^3 - 3x^{-1} + 3x - x^{1/3}$$

$$f'(x) = 3x^2 + 3x^{-2} + 3 - \frac{1}{3}x^{-2/3}$$

$$f''(x) = 6x - 6x^{-3} + \frac{2}{9}x^{-5/3}$$

b) $y(x) = \frac{x^3 - 1}{x^2 + x}$ $\qquad y'(x) = ?$

$$y'(x) = \frac{(3x^2)(x^2+x) - (x^3-1)(2x+1)}{(x^2+x)^2}$$

c) $g(t) = (t^2+1)(t^3-4t)^2$ $\qquad g'(t) = ?$

$$g'(t) = (2t)(t^3-4t)^2 + (t^2+1)(2)(t^3-4t)(3t^2-4)$$

d) $y^2 - 2x = xy^3 + 1$ $\qquad \frac{dy}{dx} = ?$

$$2y\frac{dy}{dx} - 2 = y^3 + 3xy^2\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{y^3 + 2}{2y - 3xy^2}$$

6. Find the equation of the tangent line to the curve $f(x) = 2x^3 - 4x^2 + 1$ at the point x=1.

[4 marks]

FIND $y = mx + b$

WHERE SLOPE $m = f'(1)$

$f'(x) = 6x^2 - 8x$, $m = f'(1) = 6 - 8$
$= -2$

POINT $(x, y) = (1, f(1))$
$= (1, -1)$

FIND b: $(-1) = (-2)(1) + b \Rightarrow b = 1$

TANGENT LINE: $y = -2x + 1$

7. The position of a particle (in metres, after t seconds), is given by $f(t) = t^3 - 9t^2 + 3$
After how many seconds is the particle's velocity exactly 7 metres/second?

[4 marks]

VELOCITY = DERIVATIVE $f'(t)$
$= 3t^2 - 18t$

WHEN IS IT $7\ m/sec$?

SOLVE $7 = 3t^2 - 18t$

$0 = 3t^2 - 18t - 7$

QUADRATIC FORMULA: $t = \dfrac{18 \pm \sqrt{18^2 + 84}}{6}$

$= \dfrac{18 \pm \sqrt{408}}{6} = \dfrac{9 \pm \sqrt{102}}{3}$

≈ 6.37 AND -0.37 (IGNORE)

THE VELOCITY IS REACHED AFTER ~~6.21~~ 6.37 seconds.

8. Consider the function $f(x) = x^3 - 3x^2 - 24x + 32$ *[8 marks]*

a) Find and state all critical points of this function. Then test each point to determine if they are a local maximum or minimum, or neither.

$$f'(x) = 3x^2 - 6x - 24 \quad ; \quad f''(x) = 6x - 6$$

FIND CRITICAL POINTS:

1) $f'(x) = 0 \Rightarrow 3(x^2 - 2x - 8) = 0$

$3(x-4)(x+2) = 0 \Rightarrow x = 4,\ x = -2$

2) $f'(x)$ DNE $\Rightarrow$ NO SUCH POINTS

TEST: EITHER

-2 $\quad$ 4

$f'(x) > 0$ $\Downarrow$ $f'(x) < 0$ $\Downarrow$ $f'(x) > 0$

MAX $\quad$ MIN

OR

$f''(4) > 0$ SO $x = 4$ IS A LOCAL MIN

$f''(-2) < 0$ SO $x = -2$ IS A LOCAL MAX

b) The point (x,y)=(1,6) is an inflection point of f(x). (you do not need to verify this)

Sketch a graph of f(x) that accurately shows the y-intercept, all extreme points, and the inflection point. (Note: you do not need to find the x-intercepts)

x	y
0	32
4	-48
-2	~~4~~60