Math 102 Sample Midterm #2

1. Solve the given equations and inequalities:

a) $x^2 - 3 - 2x = 0$ b) $\frac{3}{x-1} + \frac{1}{x+1} = 1$ c) $|3 - 5x| < 2$

2. If $f(x) = x^2 - 2x$, evaluate and simplify $f(x) - 2f(x+1) - x$

3. Find and state the domain of the function $f(x) = \sqrt{3 - x^2}$

4. Sketch a graph of the following split-definition function:

$$g(x) = \begin{cases} 2-(x+2)^2 & \text{if} \quad x < -1 \\ -\frac{1}{2}x + \frac{1}{2} & \text{if} \quad -1 \le x \le 1 \\ 2x - 1 & \text{if} \quad x > 1 \end{cases}$$

5. Complete the square to find write the equation $y=-2x^2+8x-7$ in the form $y=a(x-b)^2+c$.

6. A (very expensive) consultant charges a flat monthly payment, plus an additional fixed payment for every hour worked on a case. In September, the consultant worked 150 hours on a case and charged $37,750. In October, the consultant worked 165 hours on a case and charged $41,125.

   a) Find the equation that models the total charge C as a function of the number of hours worked x. What is the flat and per hour charge?
   b) How many hours would the consultant have to work in order to charge at least $50,000?

7. A ball is thrown vertically in the air from an initial height of 10 metres. It's height, in metres after t seconds, is given by the function $h(t) = -\frac{1}{2}t^2 + \frac{1}{2}t + 10$

   a) How far off the ground is the ball after 3 seconds?
   b) At what time will the ball hit the ground?
   c) What is the maximum height that this ball will attain?

Answers:

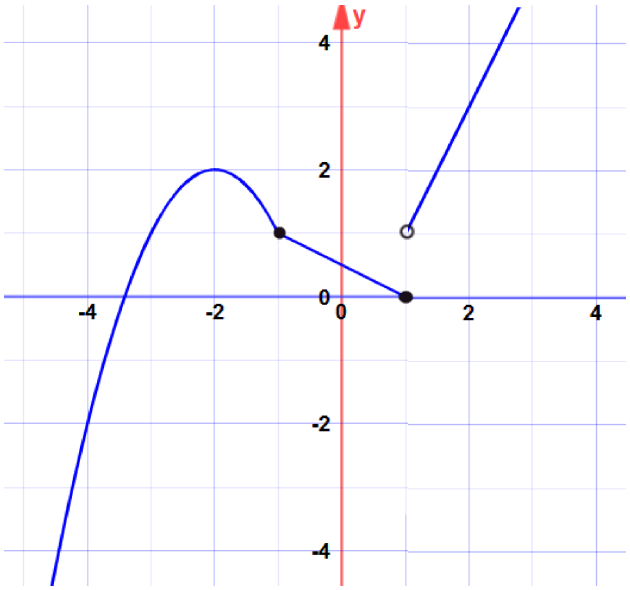
1.
a) Factor $(x-3)(x+1)=0$, so solution is $x=3$ and $x=-1$
b) Multiply by common denominator $(x-1)(x+1)$ to get $3(x+1)+1(x-1)=(x+1)(x-1)$
simplify: $x2-4x-3=0$ Use quadratic formula to get solutions $x=2\pm\sqrt{7}$
c) Solve the double inequality: $-2<3-5x<2$
to get $1/5 < x < 1$

2. $f(x)-2f(x+1)-x = (x^2-2x) - 2[(x+1)^2-2(x+1)]-x$
$= - x^2 -3x + 2$

3. For this function, we require $3-x^2 \geq 0$, i.e. $x^2 \leq 3$,
i.e. the domain is $-\sqrt{3} \leq x \leq \sqrt{3}$

4. (Note that the first function is a downward parabola with vertex (-2,2), the 2nd and 3rd function are straight lines. The first two graphs meet at (-1,1), the 3rd graph introduces a jump discontinuity).

Sketch:



5. $y = -2 (x^2 + 4x) - 7$
$= -2 (x^2 +4x + 4 - 4) -7$
$= -2 (x^2 + 4x + 4) + 8 -7$
$= -2 (x+2)^2 + 1$

Hence $a=-2$, $b=-2$, $c=1$. This parabola opens downward, with vertex $(x,y)=(-2,1)$

6.

a) This is a linear relation, find C=mx + b.

First slope m=rise/run = (37750-41125) / (150-165) = 225

Now find intercept: C=mx+b

37750 = (225)(150) + b solve for b=4000

Hence the charge is C = 225x + 4000

(i.e. (s)he charges a $4000 flat fee plus $225 per hour of work)

b) Let C=50,000, solve 50,000 < 4000 + 225 x for x>204.4

hence the consultant would need to work at least 225 hours.

7.

a) After 3 seconds, the height is h(3)=7 metres.

b) Solve h(t)=0 (either factor or use the quadratic formula)

to get intercepts t=5 and t=-4

We can disregard the negative time, hence the ball hits the ground after 5 seconds.

c) Find the vertex. Either complete the square or go half-way between the intercepts to find t=1/2 second.

The ball reaches its vertex after 0.5 seconds, the maximum height is h(0.5)=10.125 metres. (i.e. it only rises 0.125 metres before starting to fall)