

Math 102 Sample Midterm #1

1. Solve the given equations and inequalities:

a) $4x^2 - x = 3$

b) $\frac{4}{3} - 3x < \frac{1}{2}$

c) $|2x - 3| = x + 1$

2. a) Find and state the domain of $f(x) = \frac{5}{\sqrt{6-7x}}$

b) Find and state the range of $g(x) = 3 - (x - 2)^2$

c) If $h(x) = \frac{4}{x}$, simplify the expression $\frac{h(3+x) - h(3)}{x}$

3. Determine the symmetries, if any, of the equation $x^2 = yx^4 + 1$

4. The trout population in a northern Saskatchewan lake was 40 thousand fish in the year 2000. A survey in the year 2010 showed that the population had declined to just 25 thousand fish. Let N be the population size (in thousands) and t be time (in years, where $t=0$ is the year 2000).

a) If the relation between N and t is assumed to be linear, i.e. of form $N=mt+b$, find the equation that models the decline of the fish population over time.

b) What does your model predict the fish population will be in the Year 2015.

c) When does your model predict the fish population will completely die out.

5. Sketch graphs of the following equations:

a) $y + (x-2)^2 = 1$

b) $(x-2)^2 + (y+1)^2 = 9$

6. The profit function, i.e. the weekly profit P (in millions of \$) of manufacturing and selling x (thousand) items, is given by $P(x) = 40x - 0.05x^2 - 1000$

a) What is the profit of manufacturing and selling 30 thousand items?

b) Find the x -intercepts of this function. What information do these numbers give you?

c) Find the optimal production size, i.e. the value of x that will give you maximal profits.
What is that maximum attainable weekly profit?

Answers:

1.

a) Re-arrange as $4x^2 - x - 3 = 0$, i.e. $(4x + 3)(x - 1) = 0$

Solutions are $x = -3/4$ and $x = 1$.

b) $-3x < -\frac{5}{6}$ hence solution is $x > \frac{5}{18}$

c) Case I: Solve $2x - 3 = x + 1$ for $x = 4$.

Check $|8 - 3| = 4 + 1$ ✓

Case II: Solve $-(2x - 3) = x + 1$ for $x = 2/3$

Check $|4/3 - 3| = 2/3 + 1$ ✓

Hence solutions are both $x = 4$ and $x = 2/3$

2.

a) No negatives under square root, and no zero in division. Solve $6 - 7x > 0$

Hence the domain is all $x < 6/7$

b) Note that $g(x)$ is a downward opening parabola with vertex at $(x, y) = (2, 3)$.

Hence no y -value above $y = 3$ can be attained. Hence the range is all $y \leq 3$

c) $\frac{\frac{4}{3+x} - \frac{4}{3}}{x} = \frac{\frac{12 - 12 - 4x}{(3+x)(3)}}{4} = \frac{-4}{9 + 3x}$

3.

Replace x with $(-x)$ to get $x^2 = y^4 + 1$ this graph is symmetric across the y -axis

Replace y with $(-y)$ to get $x^2 = -yx^4 + 1$ this graph is not symmetric across the x -axis

Replace both x and y to get $x^2 = -yx^4 + 1$... this graph is not symmetric across the origin.

4.

a) Find slope = Rise/Run = $(25 - 40)/(10 - 0) = -3/2$

Find intercept b : Use $t = 0$ to immediately see that $b = 40$.

Hence the equation is $N = 40 - 3/2 t$

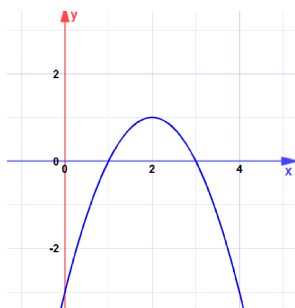
b) Let $t = 15$, then $N = 40 - 3/2 (15) = 17.5$ (thousand) fish

c) Let $N = 0$, solve $0 = 40 - 3/2 t$ for $t = 26.6$ years (i.e. halfway through the year 2026)

5.

a) Rearrange to get $y = 1 - (x - 2)^2$, i.e. a downward opening parabola with vertex $(x, y) = (1, 2)$

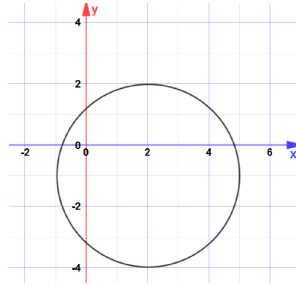
Sketch:



(Note that your graph should show intercepts correctly, i.e. x -intercepts at 1 and 3, y -intercept at -3.)

b) This is a circle with centre $(x,y)=(2,-1)$ and radius $r=3$.

Sketch:



6.

a) Evaluate $P(30) = 155$, weekly profit is \$155 million.

b) Solve $P(x)=0$ for $0 = -0.05x^2 + 40x - 1000$

Use quadratic formula to get
$$x = \frac{-40 \pm \sqrt{40^2 - 4(0.05)(1000)}}{-0.1}$$

i.e. two x-intercepts, roughly $x=25.83$ and $x=774.17$

Interpretation: The x-intercepts give us the production sizes that would result in zero profit. I.e. in order to have a positive profit, you must produce somewhere between 26 and 774 thousand items each week.

c) Find the vertex, either by completing the square, or using the midpoint of the two intercepts in part (b). Either way, we get an x-value of $x=400$ for the vertex, which gives us maximum weekly profits of $P(400)=\$7000$ (i.e. \$7 million).