# MATH 102 - Sample Final #2

1. Find and state all intercepts of $g(x) = \dfrac{x^2 - 4}{x^3 + 1}$

2. Consider $k(x) = \dfrac{x^2 - 4}{x^3}$. Is this function even, odd, both, or neither?

3. Solve for x:

   a) $x^2 - 4x = -3$ b) $|x| = 6x - 2$ c) $x^4 - x^3 + x^2 - x = 0$

   d) $4 + \log_3 x = 6$ e) $5e^{x^2+7} = 4$

4. A snail has been climbing up the wall of a deep well at a constant rate. 12 hours ago it was 12 metres down the well. 5 hours ago it was 9 metres down the well.
   a) Construct a function D(t), the depth as a function of time (in hours, t=0 being right now).
   b) What depth is the snail at right now? When will the snail escape the well?

5. An object is trapped in a changing magnetic field. It's distance from the observer (in metres, after t seconds) is given by the function $d(t) = 3t^4 - 17t^3 + 32t^2 - 20t + 100$
   Find all times at which the object is 100 metres from the observer, given that one of those times is at t=2 seconds.

6. Assume the population of a fish species is decreasing at a continuous rate, according to the equation $N(t) = N_0 e^{kt}$. The population is halved every 20 days. If there are 12,000 fish in the population now, in how many days will the population fall below 1,000?

7. An investment earns 2.6% interest, compounded daily. How long would it take for the original investment to quintuple in value?

8. A proportional income tax rate has different marginal tax rates based on your income x.
   For the first $10,000 of income, you pay no taxes. You pay 25% on the next $40,000 of income. You pay 40% on any income above $50,000.
   Construct a split-definition function T(x), the total taxes paid on an income of x. What would your total tax bill be if your income was $62,000?

9. Sketch graphs of the following functions:

   a) $f(x) = -\dfrac{1}{2}x + \dfrac{7}{3}$ b) $f(x) = 2(x-2)^3 + 1$ c) $f(x) = 3 - 4\cos(x)$

10. Find and state the domain of the function $f(x) = \dfrac{1}{\tan(x) + 1}$

11. Solve the inequality $2\sin(x) - 1 > 0$, given $0 \le x \le 2\pi$

MATH 102 SAMPLE FINAL #2 SOLUTIONS

1. y-INTERCEPT: SET $x=0$ $\Rightarrow$ $y = \frac{-4}{1} = -4$

x-INTERCEPT: SET $f(x)=0$ $\Rightarrow$ SOLVE

$$0 = \frac{x^2-4}{x^3+1}$$

IE $0 = x^2-4$

IE $x = \pm 2$

2. EVALUATE $k(-x) = \frac{(-x)^2-4}{(-x)^3}$

$$= \frac{x^2-4}{-x^3} = -\frac{x^2-4}{x^3} = -k(x)$$

$\Rightarrow$ THIS FUNCTION IS ODD.

3. a) $x^2-4x+3=0$

$(x-3)(x-1)=0$

$x=3, \ x=1$

b) CASE I: $x \geq 0$

$x = 6x-2 \Rightarrow x = \frac{2}{5}$ ✓

CASE II: $x<0$

$-x = 6x-2 \Rightarrow x = \frac{2}{7}$ ✓

c) $x(x^2-1)(x-1)=0$

$x=0, \ x=1, \ x=-1$

d) $\log_3 x = 2$

$x = 9$

e) $e^{x^2+7} = \frac{4}{5}$

$x^2+7 = \ln\left(\frac{4}{5}\right) \Rightarrow x = \pm\sqrt{\ln\left(\frac{4}{5}\right)-7}$

4. a) $D(t) = mt + b$ (LINEAR)

AT $t = -12$, $D = 12$

AT $t = -5$, $D = 9$

SLOPE $m = \dfrac{12 - 9}{(-12) - (-5)} = -\dfrac{3}{7}$

INTERCEPT : USE $(t, D) = (-12, 12)$

$\rightarrow \quad 12 = \left(-\frac{3}{7}\right)(-12) + b \Rightarrow b = \frac{48}{7}$

$\Rightarrow$ FUNCTION $D(t) = -\frac{3}{7}t + \frac{48}{7}$

b) RIGHT NOW $(t=0)$ THE SNAIL IS AT DEPTH $\frac{48}{7}$ m

$(\approx 6.86 \text{ m})$

TO ESCAPE, SET $D = 0$

AND SOLVE $0 = -\frac{3}{7}t + \frac{48}{7} \Rightarrow t = 16$

THE SNAIL WILL ESCAPE 16 HOURS FROM NOW

5. SOLVE $100 = 3t^4 - 17t^3 + 32t^2 - 20t + 100$

IE $0 = 3t^4 - 17t^3 + 32t^2 - 20t$

LONG DIVISION:

$$
\begin{array}{r}
3t^3 - 11t^2 + 10t \\
(t-2)\overline{)\,3t^4 - 17t^3 + 32t^2 - 20t} \\
\underline{3t^4 - 6t^3 \qquad\qquad\qquad} \\
-11t^3 \qquad\qquad \\
\underline{-11t^3 + 22t^2 \qquad} \\
10t^2 \qquad \\
10t^2 - 20t
\end{array}
$$

THE TIMES ARE

$t = 2$,

$t = 0$,

$t = 5/3$

AND FACTOR $3t^3 - 11t^2 + 10t = t(3t-5)(t-2)$

6. $N(t) = N_0 e^{kt}$ → FIND $k$ FIRST.

HALVED EVERY 20 DAYS:

$$\frac{1}{2} N_0 = N_0 e^{20k} \Rightarrow k = \frac{\ln\left(\frac{1}{2}\right)}{20}$$

NOW SOLVE FOR $t$:

$$1000 = 12\,000\, e^{kt}$$

$$\Rightarrow t = \frac{\ln\left(\frac{1}{12}\right)}{k} = \frac{\ln\left(\frac{1}{12}\right) \cdot 20}{\ln\left(\frac{1}{2}\right)}$$

$$\approx 71.7 \text{ DAYS.}$$

7. $B(t) = P\left[1 + \frac{.026}{365}\right]^{365t}$

SOLVE $5 = \left[1 + \frac{.026}{365}\right]^{365t}$

FOR $\ln(5) = 365t \cdot \ln\left[1 + \frac{.026}{365}\right]$

$$t = \frac{\ln(5)}{365 \cdot \ln\left[1 + \frac{.026}{365}\right]} \approx 61.9 \text{ YEARS}$$

8. FOR $0 \le x < 10{,}000$, $T(x) = 0$.

FOR $10{,}000 \le x < 50{,}000$ $\quad T(x) = .25(x - 10000)$

$$= .25x - 2500$$

FOR $x \ge 50{,}000$ $\quad T(x) = \underbrace{10000}_{\text{TAX PAID UP TO \$50,000}} + \cancel{k}\,.4(x - 50000)$

$$= .4x - 10000$$

SO

$$T(x) = \begin{cases} 0 & \text{IF } 0 \le x < 10000 \\ .25x - 2500 & \text{IF } 10000 \le x < 50\,000 \\ .4x - 10000 & \text{IF } x \ge 50\,000 \end{cases}$$

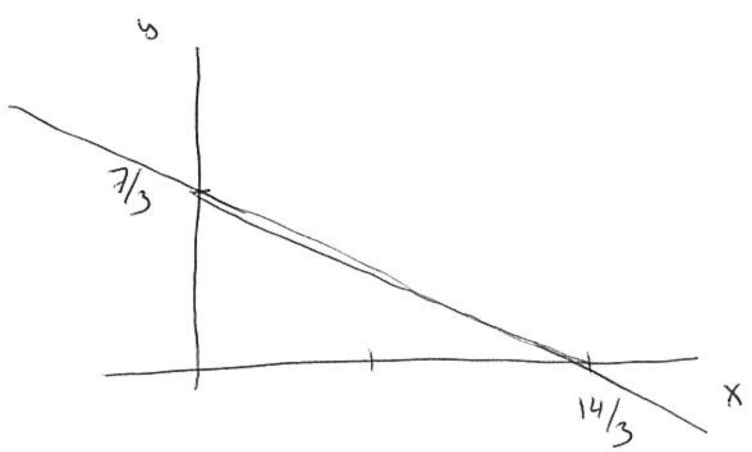
AND $T(62000) = .4(62000) - 10000$

$$= \$14\,800$$

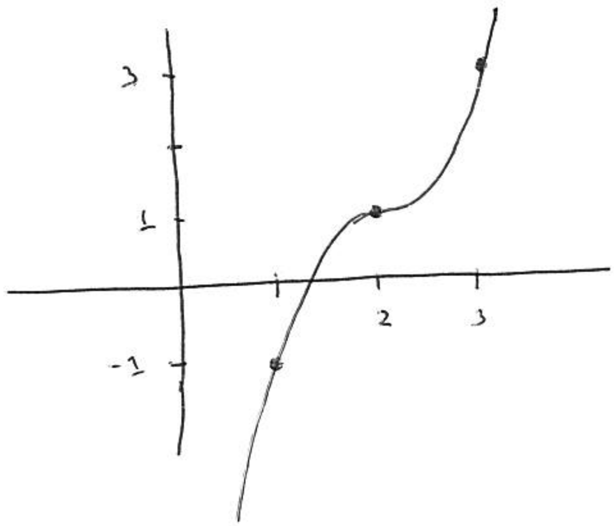
9. a) STRAIGHT LINE,

INTERCEPTS $y = 7/3$

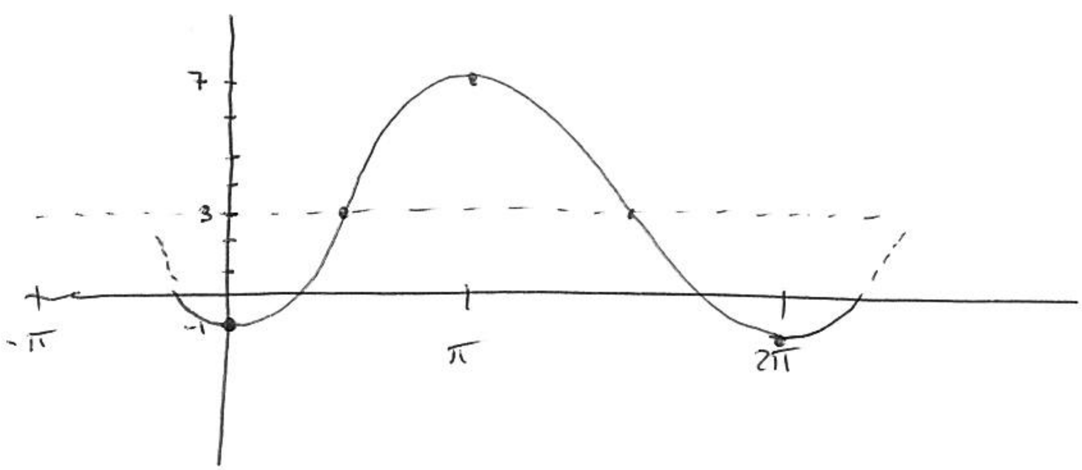
AND $x = 14/3$



b) $y = x^3$, SHIFTED 2 RIGHT
SCALED $\times 2$
SHIFTED 1 UP




c) $y = \cos x$, FLIPPED VERTICALLY, AMPLITUDE 4, SHIFT UP 3.



10. DOMAIN OF $\tan(x)$ IS ALL $x$ EXCEPT

$$x = \frac{\pi}{2},\ \frac{3\pi}{2},\ +2k\pi,\ k\in I$$

ALSO: DIVISION BY $1+\tan(x)$

CHECK $1+\tan(x)=0$

$\Rightarrow \tan(x) = -1$

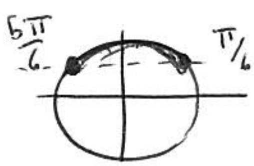
$\Rightarrow x = \frac{3\pi}{4},\ \frac{7\pi}{4},\ +2k\pi,\ k\in I$

THE DOMAIN OF $f(x)$ IS ALL $x$

EXCEPT $x = \frac{\pi}{2},\ \frac{3\pi}{4},\ \frac{3\pi}{2},\ \frac{7\pi}{4},\ +2k\pi,\ k\in I$

11. SOLVE $2\sin(x)-1=0$

$\Rightarrow \sin(x) = 1/2 \Rightarrow x = \frac{\pi}{6},\ \frac{5\pi}{6}$
$+2k\pi$




HENCE $2\sin x - 1 > 0$

FOR x-VALUES

$$\frac{\pi}{6} < x < \frac{5\pi}{6}$$

(+ ALL REPEATED INTERVALS $+2k\pi,\ k\in I$)