

M102 REVIEW #10 SOLUTIONS

1. EXPONENTIAL: $N(t) = N_0 e^{kt}$

HALF-LIFE 167 DAYS: $\frac{1}{2} N_0 = N_0 e^{167k}$

SOLVE $k = \frac{\ln(\frac{1}{2})}{167}$

NOW FIND t SUCH THAT 1% REMAINS:

SOLVE $0.01 N_0 = N_0 e^{\frac{\ln(\frac{1}{2})}{167} t}$

FOR $\ln(0.01) = \frac{\ln(\frac{1}{2})}{167} t$

$$t = \frac{167 \ln(0.01)}{\ln(0.5)} \approx 1109.5$$

IT WILL TAKE ABOUT 1110 DAYS.

2. CAN'T DIVIDE BY ZERO:

$$1 - \tan x \neq 0 \Rightarrow 1 - \frac{\sin x}{\cos x} \neq 0$$

$$\sin x \neq \cos x$$

$$\text{IE } x \neq \frac{\pi}{4} \text{ AND } x \neq \frac{5\pi}{4}$$

ALSO $\tan x$ HAS RESTRICTIONS: $x \neq \frac{\pi}{2}, x \neq \frac{3\pi}{2}$

HENCE DOMAIN IS ALL x EXCEPT

$$x = \frac{\pi}{4}, \frac{\pi}{2}, \frac{5\pi}{4}, \frac{3\pi}{2}$$

M102 #10 (CONTINUED)

#3 . $f(-x) = (-x)^3 \sin(-x) - 2 \cos(-x)$

NOTE : $(-x)^3 = -x^3$ (ODD)

$\sin(-x) = -\sin(x)$ (ODD)

$\cos(-x) = \cos(x)$ (EVEN)

SO $f(-x) = x^3 \sin x - 2 \cos x$

$= f(x) \Rightarrow$ THE FUNCTION
IS EVEN

#4 a) SOLVE $h(t) = 0$

$$0 = 40 - 5t^2 \Rightarrow 40 = 5t^2$$

$$t = \sqrt{8}$$

IT HITS THE GROUND AFTER $\sqrt{8}$ seconds
(≈ 2.83 seconds)

b) VERTEX $\rightarrow$ MIDPOINT OF INTERCEPTS $t=0$ AND $t=\sqrt{8}$

$$\rightarrow t = \frac{\sqrt{8}}{2} = \sqrt{2} \text{ seconds}$$

INSERT $t = \sqrt{2}$ TO FIND MAXIMUM HEIGHT

$$h(\sqrt{2}) = 40 - 5 \cdot (\sqrt{2})^2 = 30 \text{ metres}$$