Wednesday April 8th

Name: SOLUTIONS

ID #:

Math 102-001 Winter 2015 Quiz #5

1. The size of a bacteria population N (in millions of cells) after t hours is given by the exponential function $N(t) = N_0 e^{rt}$, where N_0 is the initial population size and r is the growth rate. *[10 marks]*

a) After three hours, the population size has grown to five times its initial size. Find the growth rate r (round to four decimal places).

SOLVE $5N_0 = N_0 e^{r \cdot 3}$ FOR r:

$$\ln 5 = 3r$$

$$r = \frac{\ln 5}{3}$$

$$\approx .5365$$

b) At the start of the experiment, the population consisted of 4 million cells. In how many hours will the population size reach 40 million? Round your time to the nearest minute.

SOLVE $40 = 4e^{\frac{\ln 5}{3} t}$ FOR t:

$$\ln 10 = \frac{\ln 5}{3} t$$

$$t = \frac{3 \cdot \ln 10}{\ln 5}$$

$$\approx 4.292$$

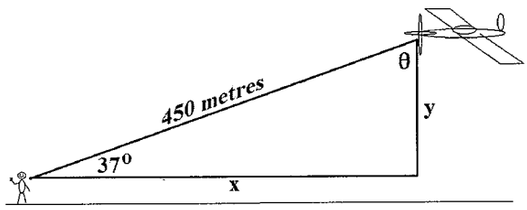
IE 4 HOURS AND 18 MINUTES.

2. Convert the given angles from degrees to radian or vice versa: *[4 marks]*

a) $22.5^\circ =$ $\pi/8$ (radian) b) $4\pi/9 =$ 80° (degrees)

3. A (crudely drawn) farmer is standing on a flat field, and notices an (aerodynamically challenged) airplane in the distance. The angle of elevation is 37°, and the direct distance (nose-to-nose) from farmer to airplane is 450 metres. *[6 marks]*

How high is the airplane flying above the ground ("y")? What is the size of the angle at the tip of the airplane ("θ")? How far is the distance on the ground from farmer to airplane ("x")?



y: $\sin(37^\circ) = \frac{y}{450} \Rightarrow y = 450 \sin(37^\circ) \approx 270.8$ m

x: $\cos(37^\circ) = \frac{x}{450} \Rightarrow x = 450 \cos(37^\circ) \approx 359.4$ m

θ: $37^\circ + \theta + 90^\circ = 180^\circ$

so $\theta = 53^\circ$