Monday, March 9th

Name: SOLUTIONS

ID #:

Math 102-001 Winter 2015 Midterm

1. Solve the following equations and inequalities: *[10 marks]* 9

a) $2x-3>7x+1$

$-3>5x+1$

$-4>5x \quad \Rightarrow \quad x<-\frac{4}{5}$

b) $2x^2-9x=5$

$2x^2-9x-5=0$

$(2x+1)(x-5)=0 \quad \Rightarrow \quad x=-\frac{1}{2},\ x=5$

(OR USE QUADRATIC FORMULA)

c) $\frac{2}{x-1}=1+\frac{2}{x+1}$ MULTIPLY BY $(x-1)(x+1)=x^2-1$

$2(x+1)=(x^2-1)+2(x-1)$

$2x+2=x^2-1+2x-2$

$x^2=5 \quad \Rightarrow \quad x=\pm\sqrt{5}$

d) $|1-x|=3x+5$

CASE I: $1-x=3x+5$

$-4x=4$

$x=-1$

CHECK: $|1-(-1)|=3(-1)+5$ ✓

$2=2$

CASE II: $x-1=3x+5$

$-2x=6$

$x=-3$

CHECK: $|1-(-3)|=3(-3)+5$

$4=-4$ ×

$\Rightarrow$ ONLY ONE SOLUTION $x=-1$

2. An art object was purchased in the year 2000. Assume its value grows at a constant rate: in 2006, the object was worth \$65,000. In 2010, the object was worth \$75,000. *[6 marks]*

a) Find the value V of the object t years after its purchase. Give your answer in the form $V = mt + b$

GIVEN TWO POINTS, $(t, V) = (6, 65000)$ AND $(10, 75000)$.

SLOPE $m = \frac{\text{RISE}}{\text{RUN}} = \frac{10000}{4} = 2500$

INTERCEPT: SOLVE $65000 = (2500)(6) + b$ FOR $b = 50{,}000$

HENCE $V = 2500\,t + 50{,}000$

b) In what year will the object's value be double its original year 2000 purchase price?

2000 PURCHASE PRICE WAS \$50,000

HENCE SOLVE

$$100{,}000 = 2500\,t + 50{,}000$$

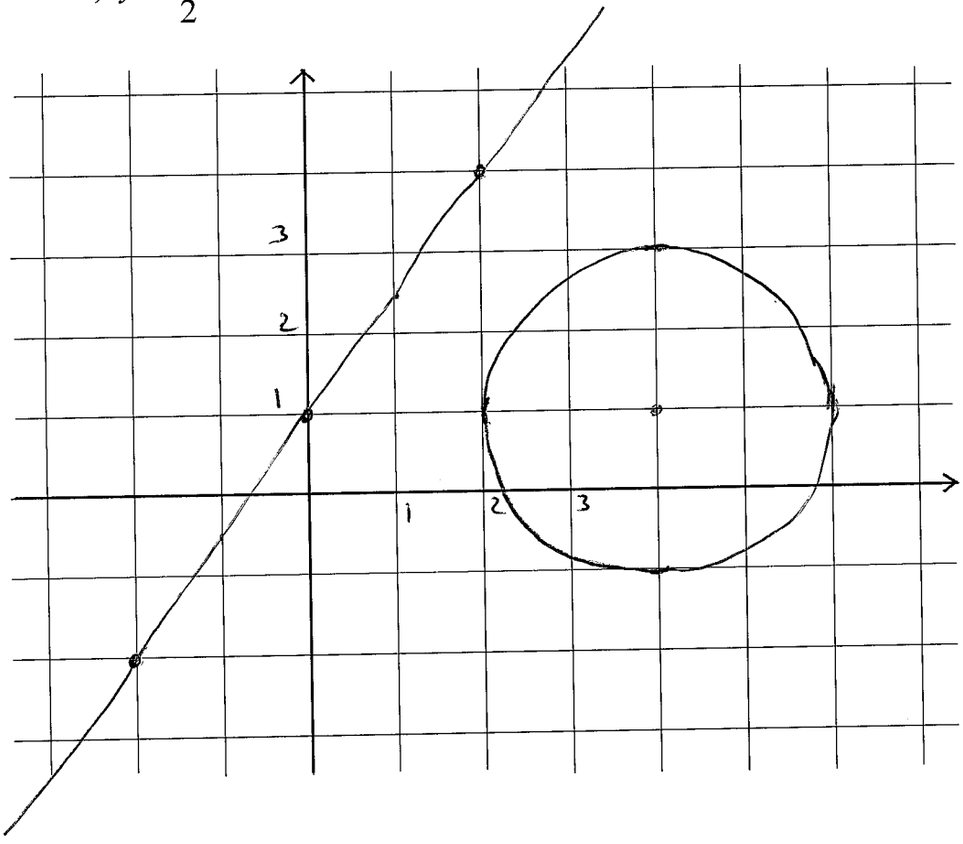
FOR $t = \frac{50{,}000}{2500} = 20$

$\Rightarrow$ IN THE YEAR 2020.

3. On the given coordinate system, sketch the graphs of both equations: *[4 marks]*

a) $y = \frac{3}{2}x + 1$ → LINE

b) $(x-4)^2 + (y-1)^2 = 4$ → CIRCLE CENTRE $(4,1)$ RADIUS 2



4. *[8 marks]*

a) Find the domain of the function $f(x)=\dfrac{x}{\sqrt{4-x^2}}$ ← DIVISION AND ROOT

NEED $4-x^2>0$

$x^2<4 \Rightarrow$ DOMAIN $-2<x<2$

b) Find all x-values at which $f(x)=x^2-x-1$ intersects $g(x)=x+3$

SOLVE $x^2-x-1=x+3$

$x^2-2x-4=0$

USE QUADRATIC FORMULA:

$$x=\frac{2\pm\sqrt{4+16}}{2}=1\pm\sqrt{5}$$

c) Given the function $f(x)=\dfrac{3}{2x-1}$, evaluate and simplify:

i) $f(-\frac{2}{3})$ $= \dfrac{3}{2(-\frac{2}{3})-1} = \dfrac{3}{-\frac{4}{3}-1} = \dfrac{3}{-\frac{7}{3}} = -\dfrac{9}{7}$

ii) $\dfrac{f(x+1)-f(1)}{x}$ $= \dfrac{\frac{3}{2(x+1)-1}-\frac{3}{2(1)-1}}{x}$

$= \dfrac{\frac{3}{2x+1}-3}{x}$

$= \dfrac{\frac{3-3(2x+1)}{2x+1}}{x}$

$= \dfrac{\frac{-6x}{2x+1}}{x}$

$= \dfrac{-6}{2x+1}$

5. A manufacturer determines that the monthly profit ("P", in millions of dollars) of producing and selling "x" (thousand) items is given by the function

$$P(x) = -x^2 + 8x - 10$$

Complete the square to write the function in the form $P(x) = a(x-b)^2 + c$

What production size "x" provides the company with the maximum possible profit? What is that maximum profit? *[5 marks]*

$$P(x) = -(x^2 - 8x) - 10$$

$$= -(x^2 - 8x + 16 - 16) - 10$$

$$= -(x^2 - 8x + 16) + 16 - 10$$

$$= -(x-4)^2 + 6$$

A PRODUCTION OF 4 THOUSAND UNITS CREATES A MAXIMUM PROFIT OF $6 MILLION.

6. True or False: (circle your answer - no explanation is necessary) *[3 marks]*

a) The expression $\dfrac{\sqrt{x^2 y^6}}{x^3 y^{-1}}$ can be simplified as $(x^{-1}y)^2$ T / (F)

$$= \frac{xy^3}{x^3 y^{-1}} = x^{-2} y^{-4} = (x^{-1} y^{-2})^2 \neq$$

b) The triangle with side lengths 10, 24, and 26 contains a 90° angle (T) / F

NOTE $10^2 + 24^2 = 26^2$

c) A six sided cube (i.e. all sides are equal) with volume V=27 m^3 has a surface area of A=54 m^2 (T) / F

IF $V = 27$ THEN $x^3 = 27$ OR $x = 3$

HENCE $A = 6 \cdot x^2$

$= 6 \cdot 3^2 = 54$