Math 302.102 Fall 2010
Computing Probabilities for Discrete Random Variables

Example. Suppose that the joint probability function $\mathbf{P}\{X = x, Y = y\}$ of X and Y is as follows.

$X \downarrow Y \rightarrow$	$Y = 2$	$Y = 4$	$Y = 6$
$X = 1$	0.05	0.14	0.10
$X = 2$	0.25	0.10	0.02
$X = 3$	0.15	0.17	0.02

(a) Determine $\mathbf{P}\{XY < 6\}$.

(b) Determine the marginal for Y. In other words, determine $\mathbf{P}\{Y = y\}$ for all values of y.

(c) Determine the conditional probability function (or conditional mass function or conditional density) for X given $Y = 4$. In other words, determine $\mathbf{P}\{X = x \,|\, Y = 4\}$ for all values of x.

(d) Compute $\mathbb{E}(X \,|\, Y = 4)$, the conditional expectation of X given $Y = 4$.

(e) Determine the marginal for X. In other words, determine $\mathbf{P}\{X = x\}$ for all values of x.

(f) Determine the conditional probability function (or conditional mass function or conditional density) for Y given $X = 3$. In other words, determine $\mathbf{P}\{Y = y \,|\, X = 3\}$ for all values of y.

(g) Compute $\mathbb{E}(Y \,|\, X = 3)$, the conditional expectation of Y given $X = 3$.

(h) Compute $\mathrm{Cov}(X, Y)$, the covariance of X and Y. Note that

$$\mathrm{Cov}(X, Y) = \mathbb{E}(XY) - \mathbb{E}(X)\mathbb{E}(Y).$$

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Computing Probabilities for Continuous Random Variables

Example. Suppose that a random vector (X, Y) has joint density function

$$f_{X,Y}(x, y) = \begin{cases} 15x^2y, & \text{if } 0 < x < y < 1, \\ 0, & \text{otherwise.} \end{cases}$$

(a) Verify that $f_{X,Y}$ is, in fact, a legitimate density function.

(b) Find $f_X(x)$, the marginal density function of X.

(c) Use your result of **(b)** to compute $\mathbb{E}(X)$.

(d) Find $f_{Y|X=x}(y) = f_{Y|X}(y|x)$, the conditional density function of Y given $X = x$.

(e) Compute $\mathbb{E}(Y \mid X = x)$.

(f) Find $f_Y(y)$, the marginal density function of Y.

(g) Use your result of **(f)** to compute $\mathbb{E}(Y)$.

(h) Compute $\text{Cov}(X, Y)$, the covariance of X and Y. Note that

$$\text{Cov}(X, Y) = \mathbb{E}(XY) - \mathbb{E}(X)\mathbb{E}(Y).$$