Math 302.102 Fall 2010
Summary of Continuous Random Variables

Example 1. The random variable X has an *exponential distribution with parameter* $\lambda > 0$ if the density of X is

$$f(x) = \begin{cases} \lambda e^{-\lambda x}, & x \geq 0, \\ 0, & x < 0. \end{cases}$$

The distribution function of X is

$$F(x) = \begin{cases} 1 - e^{-\lambda x}, & x \geq 0, \\ 0, & x < 0. \end{cases}$$

We often write $X \sim \mathrm{Exp}(\lambda)$ for such a random variable.

Example 2. The random variable X has a *uniform distribution on* $[a, b]$, $-\infty < a < b < \infty$, if the density of X is

$$f(x) = \begin{cases} \dfrac{1}{b-a}, & a \leq x \leq b, \\ 0, & \text{otherwise.} \end{cases}$$

The distribution function of X is

$$F(x) = \begin{cases} 0, & x < a, \\ \dfrac{x-a}{b-a}, & a \leq x \leq b, \\ 1, & x > b. \end{cases}$$

We often write $X \sim \mathrm{Unif}(a, b)$ for such a random variable.

Example 3. The random variable X has a *Cauchy distribution with parameter* $\theta \in \mathbb{R}$ if the density of X is

$$f(x) = \frac{1}{\pi} \cdot \frac{1}{1 + (x - \theta)^2}$$

for $-\infty < x < \infty$. The distribution function of X is

$$F(x) = \frac{1}{2} + \frac{1}{\pi} \arctan(x - \theta)$$

for $-\infty < x < \infty$. We often write $X \sim \mathrm{Cauchy}(\theta)$ for such a random variable.

Example 4. The random variable X has a *Normal distribution with parameters* $\mu \in \mathbb{R}$ *and* $\sigma > 0$ if the density of X is

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left\{ -\frac{(x-\mu)^2}{2\sigma^2} \right\}$$

for $-\infty < x < \infty$. The distribution function of X is

$$F(x) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{x} \exp\left\{ -\frac{(t-\mu)^2}{2\sigma^2} \right\} \mathrm{d}t$$

for $-\infty < x < \infty$.

Note that there is no closed-form expression for the distribution function of a normal random variable. In order to evaluate $F(x)$ for a particular x it is necessary to resort to a numerical approximation. This is why tables of normal probabilities have been compiled. Also note that sometimes Φ is used for the normal distribution function so that $\Phi(x) = F(x)$. We often write $X \sim \mathcal{N}(\mu, \sigma^2)$ for such a random variable.

Example 5. The random variable X has a *Gamma distribution with parameters* $\alpha > 0$ *and* $\lambda > 0$ if the density of X is

$$f(x) = \begin{cases} \dfrac{\lambda^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\lambda x}, & x \geq 0, \\ 0, & x < 0. \end{cases}$$

Here, Γ is the gamma function defined by

$$\Gamma(\alpha) = \int_0^\infty x^{\alpha-1} e^{-x} \, \mathrm{d}x.$$

We often write $X \sim \text{Gamma}(\alpha, \lambda)$ for such a random variable. The distribution function of X cannot be evaluated in closed-form and so it is rarely used.

Let $n = 1, 2, \ldots$. We sometimes say that the random variable X has a *Chi-square distribution with* n *degrees of freedom* and write $X \sim \chi^2(n)$ if $X \sim \text{Gamma}(1/2, n/2)$.

Also note that $X \sim \text{Exp}(\lambda)$ if and only if $X \sim \text{Gamma}(1, \lambda)$.

Example 6. The random variable X has a *Beta distribution with parameters* $a > 0$ *and* $b > 0$ if the density of X is

$$f(x) = \begin{cases} \dfrac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} x^{a-1}(1-x)^{b-1}, & 0 \leq x \leq 1, \\ 0, & \text{otherwise.} \end{cases}$$

We often write $X \sim \text{Beta}(a, b)$ for such a random variable. The distribution function of X cannot be evaluated in closed-form and so it is rarely used.

Note that $X \sim \text{Unif}(0, 1)$ if and only if $X \sim \text{Beta}(1, 1)$.