Math 302.102 Fall 2010
Prerequisite Review Exercises

Math 302 provides an introduction to the mathematical theory of probability. The tools that we will use in this class include integration-by-substitution and integration-by-parts from first year calculus, and multiple integration from second year calculus. The beginning of the term is a good time to review this material. We will not be reviewing techniques of integration when we encounter integrals in a few weeks. Here are a couple of direct computations to help you refresh your skills, and to illustrate the type of integrals we will need to consider.

Single-Variable Calculus

1. Compute $\displaystyle\int_0^\infty e^{-2x}\,\mathrm{d}x$.
2. Compute $\displaystyle\int_0^\infty xe^{-2x}\,\mathrm{d}x$.
3. Compute $\displaystyle\int_0^\infty x^2e^{-2x}\,\mathrm{d}x$.
4. Compute $\displaystyle\int_0^\infty x^3e^{-2x}\,\mathrm{d}x$.
5. Compute $\displaystyle\int_0^\infty x^{-2/3}e^{-x^{1/3}}\,\mathrm{d}x$.
6. For any $a > 0$, compute $\displaystyle\int_0^\infty x^{1/a-1}e^{-x^{1/a}}\,\mathrm{d}x$.
7. Compute $\displaystyle\int_0^\infty x^{1/3}e^{-2x^{1/3}}\,\mathrm{d}x$.
8. Compute $\displaystyle\int_0^\infty xe^{-x^2}\,\mathrm{d}x$.
9. For any $a > 0$, compute $\displaystyle\int_0^\infty xe^{-ax^2}\,\mathrm{d}x$.
10. Compute $\displaystyle\int_{-\infty}^\infty x^2e^{-x^2}\,\mathrm{d}x$.
11. For any $a > 0$, compute $\displaystyle\int_{-\infty}^\infty x^2e^{-ax^2}\,\mathrm{d}x$.
12. Compute $\displaystyle\int_0^1 x(1-x)^3\,\mathrm{d}x$.
13. Compute $\displaystyle\int_0^1 x^2(1-x)^3\,\mathrm{d}x$.
14. Compute $\displaystyle\int_{-\infty}^\infty \frac{1}{x^2+1}\,\mathrm{d}x$.

15. Compute $\displaystyle\int_0^\infty \frac{x}{x^2+1}\,\mathrm{d}x$.

16. Compute $\displaystyle\int_{-\infty}^\infty \frac{x}{x^2+1}\,\mathrm{d}x$.

17. For any $a > 0$, compute $\displaystyle\int_a^\infty \frac{1}{x^3}\,\mathrm{d}x$.

18. For any $a > 0$ and for any $b > 1$, compute $\displaystyle\int_a^\infty \frac{1}{x^b}\,\mathrm{d}x$.

Multi-Variable Calculus

For each of the following, compute the value of

$$\iint_R f(x,y)\,\mathrm{d}x\,\mathrm{d}y$$

where R is the region indicated.

1. $f(x,y) = x^2$ and $R = \{0 < x < y < 1\}$.
2. $f(x,y) = x^2$ and $R = \{0 < y < x < 1\}$.
3. $f(x,y) = y^2$ and $R = \{0 < x < y < 1\}$.
4. $f(x,y) = xy$ and $R = \{0 < y < x < 1\}$.
5. $f(x,y) = x + y$ and $R = \{0 < x < y < 1\}$.
6. $f(x,y) = e^{-2y}$ and $R = \{0 < x < 2y\}$.
7. $f(x,y) = \sqrt{x^2+y^2}$ and $R = \{x^2 + y^2 \le 1\}$. (Hint: use polar coordinates)
8. $f(x,y) = xy$ and $R = \{x^2 + y^2 \le 1,\ x > 0,\ y > 0\}$. (Hint: use polar coordinates)

Some Sums

1. Compute the exact value of $\displaystyle\sum_{j=0}^\infty 3^{-j}$. (Hint: It is a particular geometric series.)

2. Compute the exact value of $\displaystyle\sum_{j=1}^\infty j3^{-j}$. (You may not have seen this one before.)

3. Compute the exact value of $\displaystyle\sum_{j=0}^\infty \frac{3^{-j}}{j!}$. (Hint: It is a particular power series.)