Math 302.102 Fall 2010
Tips for Using Bayes' Rule

In order to use Bayes' Rule correctly is is absolutely vital that you VERY CAREFULLY define appropriate events. In the statement of the problem, you will be given a conditional probability for one event given another event, and you will want to determine a conditional probability for those events swapped. We will call these events A and B_1 so that you are given the conditional probability

$$\mathbf{P}\{A \mid B_1\}.$$

You want to determine

$$\mathbf{P}\{B_1 \mid A\}.$$

Write down the basic version of Bayes' Rule using the fact that $A \cap B_1 = B_1 \cap A$; that is, $\mathbf{P}\{A \cap B_1\} = \mathbf{P}\{B_1 \cap A\}$ and so

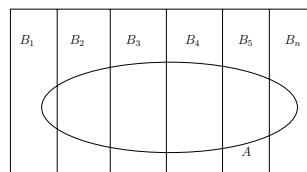$$\mathbf{P}\{A \mid B_1\}\mathbf{P}\{B_1\} = \mathbf{P}\{B_1 \mid A\}\mathbf{P}\{A\}$$

implying that

$$\mathbf{P}\{B_1 \mid A\} = \frac{\mathbf{P}\{A \mid B_1\}\mathbf{P}\{B_1\}}{\mathbf{P}\{A\}}. \qquad (*)$$

Hence, we need to determine the values of the three probabilities on the right-side of the previous equation $(*)$. As noted already, in the statement of the problem you will be given $\mathbf{P}\{A \mid B_1\}$. You will also be given $\mathbf{P}\{B_1\}$, or at least enough information to determine it, such as $\mathbf{P}\{B_1^c\}$. The key step will be to use to law of total probability to determine $\mathbf{P}\{A\}$. Here is the key. You want to *partition* the sample space. You will need to use B_1 for this The event B_1 will typically give clues to how the other events $B_2, \ldots, B_n$ should be defined. Schematically, you have the following.
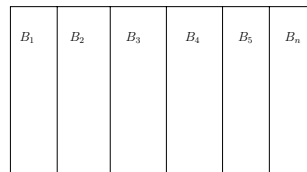
Now consider the event A.

Now, write A as a disjoint union of the ways it can intersect the partition $B_1, B_2, \ldots, B_n$, namely

$$\mathbf{P}\{A\} = \mathbf{P}\{A \cap B_1\} + \mathbf{P}\{A \cap B_2\} + \cdots + \mathbf{P}\{A \cap B_n\}$$
$$= \mathbf{P}\{A \mid B_1\}\mathbf{P}\{B_1\} + \mathbf{P}\{A \mid B_2\}\mathbf{P}\{B_2\} + \cdots + \mathbf{P}\{A \mid B_n\}\mathbf{P}\{B_n\}.$$

This is the Law of Total Probability. Notice that this expression includes the same term as the numerator, namely $\mathbf{P}\{A \mid B_1\}\mathbf{P}\{B_1\}$, but also includes more terms. Information about these other terms will also be given in the statement of the problem.

Remark. Problem #3 on Assignment #2 (the one about Elvis having a twin brother) can be solved by following this template precisely.

Remark. Sometimes a tree diagram is useful for figuring out the possible events to use for the partition $B_1, \ldots, B_n$.