Math 302.102 Fall 2010
Class Exercises from September 20, 2010

Recall that the sample space $S$ is defined as the collection of all possible outcomes of a chance experiment. An event consists of certain outcomes so that it is a subset of the sample space. A probability is a number between 0 and 1 assigned to events in such a way that

**(a)** $\mathbf{P}\{S\} = 1$, $\mathbf{P}\{\emptyset\} = 0$,

**(b)** $\mathbf{P}\{A^c\} = 1 - \mathbf{P}\{A\}$, and

**(c)** $\mathbf{P}\{A \text{ or } B\} = \mathbf{P}\{A \cup B\} = \mathbf{P}\{A\} + \mathbf{P}\{B\}$ whenever $A$ and $B$ are disjoint (i.e., mutually exclusive).

If $A$ and $B$ are not disjoint, then (look at a Venn diagram) we find

$$\mathbf{P}\{A \cup B\} = \mathbf{P}\{A\} + \mathbf{P}\{B\} - \mathbf{P}\{A \cap B\}.$$

We say that the events $A$ and $B$ are independent if

$$\mathbf{P}\{A \text{ and } B\} = \mathbf{P}\{A \cap B\} = \mathbf{P}\{A\} \cdot \mathbf{P}\{B\}.$$

Finally, we define the the conditional probability of $A$ given $B$ by setting

$$\mathbf{P}\{A \mid B\} = \frac{\mathbf{P}\{A \cap B\}}{\mathbf{P}\{B\}}$$

so long as $\mathbf{P}\{B\} > 0$. (Look at a Venn diagram.)

**Reference.** We have basically covered everything in Chapter 2 of Ross except for Section 2.6 (which is optional anyway). However, we have not done anything with combinations and permutations which is Chapter 1 of Ross. Therefore, some of the exercises and examples in Chapter 2 are not yet suitable for us. We have also covered most of Sections 3.1 and 3.2. For extra practice, you can work through the exercises at the end of each chapter. Most solutions are linked through our course website.

**Problem 1.** On any given morning, when my son wakes up, there is an 80% chance that he will be immediately thirsty. There is also a 55% chance that he will be immediately hungry. If he is either thirsty or hungry, then I must immediately get out of bed and prepare something for him. However, if he is neither thirsty nor hungry, then I can continue to stay in bed. What can you say about the probability that on any given morning I can continue to stay in bed?

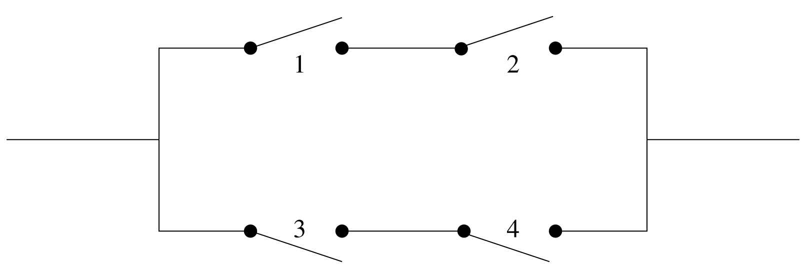
**Problem 2.** Consider the following simplified voter model. Assume that eligible BC electors cast their votes independently and according to the current proportions in the legislature, namely Liberal (0.56), New Democrat (0.41), or Other (0.03). Suppose that one eligible BC elector is selected at random. Construct a complete probability model to describe this scenario, namely specify the sample space, list all possible events, and prescribe probabilities to all possible events.

**Problem 3.** Consider the following simplified model of stock price movement. Assume that on any given day, the value of *Koz, Inc.* is likely to increase by \$1 with probability 0.56, decrease by \$1 with probability 0.41, or remain the same with probability 0.03. Assume further that the stock's movement on any day is independent of the stock's movement on any other day. Construct a complete probability model for one day's stock price movement, namely specify the sample space, list all possible events, and prescribe probabilities to all possible events.

**Problem 4.** The transmission of hereditary characteristics from parent to offspring is often called *Medelian inheritence*, named after Gregor Mendel, and is one of the core theories of classical genetics. The idea is that genes occur in pairs in ordinary body cells but segregate when sex cells are formed. There are three distinct ways that genes can be paired for each trait, namely $AA$, $Aa$, or $aa$. As is traditional in genetics, capital letters are used to identify dominant genes and lower case letters are used to identify recessive genes. Assume that in one family, the gene pairing $AA$ occurs with probability 0.41, the gene pairing $Aa$ occurs with probability 0.56, and the gene pairing $aa$ occurs with probability 0.03. Suppose that one member of this family is selected at random and this trait's gene pairing is examined. Construct a complete probability model for this scenario, namely specify the sample space, list all possible events, and prescribe probabilities to all possible events.

**Problem 5.** Suppose that the sample space $S$ consists of three outcomes, say $S = \{a, b, c\}$. Suppose further that outcome $a$ occurs with probability 0.56, outcome $b$ occurs with probability 0.41, and outcome $c$ occurs with probability 0.03. Explicitly list all of the possible events. Call this collection of events $\mathcal{F}$, and give the value of $\mathbf{P}\{A\}$ for every $A \in \mathcal{F}$.

**Problem 6.** Consider the following circuit with four switches labelled 1, 2, 3, 4. Assume that switches 1 and 2 are each closed with probability 0.60, while switches 3 and 4 are each closed with probability 0.75. Assume further that the switches function independently. Determine the probability that a current can flow from left-to-right through the circuit. (Note that in order for current to flow there must be at least one closed path from left-to-right.)




**Problem 7.** An experiment consists of first tossing a fair coin and then rolling a standard six-sided fair die. If this experiment is repeated successively, what is the probability of obtaining a heads on the coin before a 1 or 2 on the die?