Math 302.102 Fall 2010
Assignment #5

This assignment is due at the beginning of class on Wednesday, November 3, 2010.

1. One important use of Bayes' Rule is in the context of a researcher soliciting responses to *sensitive questions*; that is, questions to which a respondent is unlikely to answer truthfully due to the possibility of embarrassment or incrimination. Here is an example: *Have you ever smoked marijuana?* Instead of answering the researcher's question directly, the respondent rolls a standard six-sided die, but does not show the researcher the result of the roll. If the number 1, 2, 3, or 4 appears, then the respondent answers the sensitive question truthfully. If the number 5 or 6 appears, then the respondent lies. Hence, the respondent answers only YES or NO. Since the researcher does not see the outcome of the die roll, the researcher has no idea whether or not the respondent is answering the sensitive question truthfully. Note that "the respondent lies" means that the respondent answers NO if the true answer is YES, and answers YES if the true answer is NO. Suppose that a researcher uses this procedure to solicit responses from 100 randomly chosen people. Of those people, 40 of them answer YES.

(a) If John is among these randomly chosen people, what is the probability that John has smoked marijuana? *Hint*: Use the law of total probability to find an equation for $\mathbf{P}\{\text{YES}\}$ in terms of $\mathbf{P}\{\text{smoked marijuana}\}$ and then solve for $\mathbf{P}\{\text{smoked marijuana}\}$.

(b) If John is one of those people who answered YES, what is the probability that John has smoked marijuana?

2. A basket contains w white balls, r red balls, and b black balls. A ball is selected at random and its colour is noted. The selected ball is then replaced along with d more balls of the same colour (so that there are now $w + r + b + d$ balls in the basket). Then another ball is drawn at random from the basket.

(a) Determine the probability that the second ball drawn is black.

(b) Suppose that the second ball drawn is black. What is the probability that the first ball drawn was red?

3. Suppose that in a class of 85 students, the average assignment grade was 93%, the average midterm grade was 75%, and the average final exam grade was 80%. Suppose further that assignments count for 14% of the course grade, the midterm counts for 36% of the course grade, and the final exam counts for 50% of the course grade. Determine the average course grade for students in this class.

4. Consider the following bet which can be made in the casino game of craps. (It happens to be called a *working 6/8 place bet* but that is unimportant.) The player bets \$12. The dice are rolled repeatedly until either (i) a 7 appears (in which case the player loses the \$12 bet) or (ii) either a 6 or 8 appears (in which case the player wins \$7 in addition to the \$12 bet). Let X denote the player's net winnings for this bet. Compute both $\mathbf{P}\{X = 7\}$ and $\mathbf{P}\{X = -12\}$, and then use these values to determine the expected value of X.

5. Suppose that $X \sim \mathcal{N}(0,1)$ so that the density function of X is

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$$

for $-\infty < x < \infty$. Use a suitable Riemann sum approximation to estimate

(a) $\mathbf{P}\{0 \le X \le 0.0001\}$, and

(b) $\mathbf{P}\{1 \le X \le 1.0001\}$.

6. The distance that a tire selected at random from a used tire shop lasts for is $10000X$ kilometres, where X is a random variable with the density function

$$f(x) = \begin{cases} \frac{2}{x^2}, & \text{if } 1 < x < 2, \\ 0, & \text{otherwise.} \end{cases}$$

(a) What is the expected distance that such a randomly selected tire lasts for?

(b) Of those tires that last for less than 15000 kilometres, what fraction last for between 10000 and 12500 kilometres?

7. Suppose that X and Y are independent random variables having distributions as indicated. Compute $\mathbf{P}\{Y > X\}$. The notation is defined on the *Summary of Continuous Random Variables* handout. *Note*: Use the law of total probability for continuous random variables, but for **(c)** be careful about which random variable you condition on.

(a) $X \sim \text{Exp}(\lambda_1)$ and $Y \sim \text{Exp}(\lambda_2)$

(b) $X \sim \text{Unif}(a_1, b_1)$ and $Y \sim \text{Unif}(a_2, b_2)$

(c) $X \sim \mathcal{N}(0,1)$ and $Y \sim \text{Unif}(0,1)$

8. Suppose that X_1 and X_2 are independent random variables each having common density function

$$f(x) = \begin{cases} x \exp\left\{-\frac{x^2}{2}\right\}, & x \ge 0, \\ 0, & x < 0. \end{cases}$$

Let $Y = \min\{X_1, X_2\}$.

(a) Determine the density function of Y.

(b) Determine the density function of Y^2.

(c) Compute $E(Y^2)$.