Stat 862: Assignment #6

Problem #1: Suppose that $B = \{B_t,\ t \geq 0\}$ is a standard Brownian motion with $B_0 = 0$. For fixed t and s, determine the distribution of the random variable $X = B_t + B_s$.

Problem #2: Suppose that $B = \{B_t,\ t \geq 0\}$ is a standard Brownian motion with $B_0 = 0$.

(a) Compute the conditional probability

$$P(B_2 > 0 | B_1 > 0).$$

Hint: Use the Chapman-Kolmogorov equation, and compute the resulting double integral with polar coordinates.

(b) Are the events $\{B_2 > 0\}$ and $\{B_1 > 0\}$ independent?

Problem #3: Suppose that $B = \{B_t,\ t \geq 0\}$ is a standard Brownian motion with $B_0 = 0$, and let

$$X = \int_0^1 (2 - t)\, dB_t \quad \text{and} \quad Y = \int_0^1 (3 - 4t)\, dB_t.$$

(a) Show that both X and Y are identically distributed random variables. *Hint: Show X and $Y \sim \mathcal{N}(0, 7/3)$.*

(b) Are X and Y independent? *Hint: Compute* $\mathrm{Cov}(X, Y)$.

Problem #4: Suppose that $B = \{B_t,\ t \geq 0\}$ is a standard Brownian motion with $B_0 = 0$. Determine the distribution of

$$X = \int_0^1 B_t \cos(1 - t)\, dt.$$

Problem #5: Suppose that $B = \{B_t,\ t \geq 0\}$ is a standard Brownian motion with $B_0 = 0$. Find all constants a and b such that the Wiener integral

$$I_t = \int_0^t \left(a + \frac{bs}{t}\right) dB_s$$

is also a Brownian motion.