Stat 852 Fall 2011
Assignment #5

This assignment is due at the beginning of class on Monday, December 5, 2011.

1. Let $X_1, \ldots, X_n$ be iid random variables with probability mass function

$$P_\mu(X_1 = x) = \frac{e^{-\mu}\mu^x}{x!}, \quad x \in \{0, 1, \ldots\},\ \mu > 0.$$

Consider the following estimators of $\theta = e^{-\mu}$:

(i) $T_{1,n} = e^{-T/n}$ (the MLE),

(ii) $T_{2,n} = \frac{1}{n}\sum_{i=1}^n I(X_i = 0)$, and

(iii) $T_{3,n} = (1 - \frac{1}{n})^T$

where

$$T = \sum_{i=1}^n X_i.$$

(a) Show that $T_{3,n}$ is the minimum variance unbiased estimator of θ.

(b) For $i = 1, 2$, is $T_{i,n}$ an unbiased estimator of θ? If not, is it asymptotically unbiased as $n \to \infty$? Justify your answers.

(c) For $i = 1, 2, 3$, find the asymptotic distribution of

$$\sqrt{n}(T_{i,n} - \theta)$$

as $n \to \infty$. *Hint*: For $i = 3$, first consider the statistic $\log T_{3,n}$.