Stat 851 Winter 2008
Assignment #8

This assignment is due on **Monday, April 7, 2008**.

1. Suppose that $X_1, X_2, \ldots$ are random variables defined on a common probability space. Suppose further that X_n, $n = 1, 2, 3, \ldots$, has density function

$$f_n(x) = \begin{cases} 1 - \cos(2\pi n x), & \text{if } 0 \le x \le 1, \\ 0, & \text{otherwise.} \end{cases}$$

Show that X_n converges in distribution to some random variable X and determine the distribution of X.

2. Let $X_{n1}, X_{n2}, \ldots, X_{nn}$ be independent random variables with a common distribution given by

$$P\{X_{nk} = 0\} = 1 - \frac{1}{n} - \frac{1}{n^2}, \quad P\{X_{nk} = 1\} = \frac{1}{n}, \quad P\{X_{nk} = 2\} = \frac{1}{n^2}$$

for $k = 1, 2, 3, \ldots, n$ and $n = 2, 3, \ldots$. Set

$$S_n = X_{n1} + X_{n2} + \cdots + X_{nn}, \quad n = 2, 3, \ldots.$$

Prove that S_n converges (as $n \to \infty$) in distribution to X where X has a Poisson(1) distribution.

3. Complete the following exercises from page 164:

- #18.7
- #18.8 (The random variable X_n has density function $f_n(x) = \frac{1}{\pi(1+x^2)}$, $-\infty < x < \infty$. Furthermore, $|X_n| \notin L^1$.)

4. Complete the following exercises from pages 114–116:

- #14.4, 14.15, 14.16

5. If X is uniformly distributed on $[0, a]$, determine $\varphi_X(u)$, the characteristic function of X.