Here is a short hint for Problem 2.9, to follow up the hint I gave in class.

In order to solve (c) and find an expression for the three parameters in this model in terms of ϕ, σ_w^2, and σ_z^2, it is (in my opinion) easiest to find two expressions for the ACF of $\{U_t\}$.

From (a), we have $U_t = Z_t + (W_t - \phi W_{t-1})$. You must prove that this is the sum of two stationary processes. This gives an expression for γ_U. Hint: $W_t - \phi W_{t-1}$ is a MA(1) process (WHY?) and is therefore stationary.

From (b), we have $U_t = N_t + \theta N_{t-1}$ where $\{N_t\} \sim WN(0, \sigma_n^2)$ to use the notation in class. This gives another expression for γ_U.

If you equate these two expressions, you should find that

$$\sigma_z^2 + \sigma_w^2(1 + \phi^2) = \sigma_n^2(1 + \theta^2)$$

and

$$-\sigma_w^2\phi = \sigma_n^2\theta.$$

This is the expression you want that relates all the parameters. Theoretically, you *could* solve this system of equations. HOWEVER, you do not need to, it is too difficult.