Statistics 452 Midterm Exam – October 12, 2011

This exam is worth 50 points.

This exam has 3 problems and 1 numbered page.

You have 50 minutes to complete this exam. Please read all instructions carefully, and check your answers. Show all work neatly and in order, and clearly indicate your final answers. Answers must be justified whenever possible in order to earn full credit. ***Unless otherwise specified, no credit will be given for unsupported answers, even if your final answer is correct.*** *Points will be deducted for incoherent, incorrect, and/or irrelevant statements.*

You are permitted to consult your class notes and the textbook ***Statistical Inference*** *by Casella and Berger.*

Instructor: Michael Kozdron

1. (*10 points*) For a random variable X suppose that $M_X(t) = \mathbb{E}(e^{tX})$ exists for $t \in (-h, h)$ for some $h > 0$. Show that

$$\frac{d}{dt} \log M_X(t) \bigg|_{t=0} = \mathbb{E}(X)$$

and

$$\frac{d^2}{dt^2} \log M_X(t) \bigg|_{t=0} = \mathrm{Var}(X).$$

2. (*20 points*) Consider the family of Pareto distributions with density function

$$f(x|\alpha, \beta) = \begin{cases} \beta\alpha^{\beta}x^{-\beta-1}, & \text{if } x > \alpha, \\ 0, & \text{otherwise,} \end{cases} \tag{$*$}$$

where $\alpha > 0$ and $\beta > 0$ are parameters.

- **(a)** (*7 pts*) Let α be known and let β be unknown. Does the family $(*)$ form an exponential family? Justify your answer.
- **(b)** (*7 pts*) Let α be unknown and let β be known. Does the family $(*)$ form an exponential family? Justify your answer.
- **(c)** (*6 pts*) Show that for $\beta \leq 2$ the second moment of X does not exist.

3. (*20 points*) Let X_1 and X_2 be iid random variables with probability mass function given by

$$P_\theta(X_1 = x) = \begin{cases} e^{-\theta}, & \text{for } x = 0, \\ \theta e^{-\theta} & \text{for } x = 1, \\ 1 - e^{-\theta} - \theta e^{-\theta} & \text{for } x = 2, \end{cases}$$

where $\theta > 0$ is a parameter. Show that $X_1 + X_2$ is not sufficient for θ.