Stat 452 Fall 2011
Assignment #1

This assignment is due at the beginning of class on Monday, September 26, 2011.

1. Exercise 2.3 page 76

2. Exericse 2.30 (a), (b), (c) page 80

3. Exercise 2.35 (a) page 81

4. Exercise 2.36 page 81

5. Exercise 4.4 page 192

6. Exercise 4.30 page 195

7. Exercise 4.16 page 194

8. Let (X, Y) be a random vector with a uniform distribution on

$$S = \{(x, y) : x > 0,\ y > 0,\ x + y < 1\}.$$

(a) Find the marginal density $f_X(x)$ of X.

(b) Find the moment generating function of X and use it to find $\mathrm{E}(X)$ and $\mathrm{var}(X)$.

(c) For $0 < y < 1$, find the conditional density of X given $Y = y$.

(d) For $0 < y < 1$, find $\mathrm{E}(X|Y = y)$ and $\mathrm{var}(X|Y = y)$.

(e) Use the results in **(d)** to find (again) $\mathrm{E}(X)$ and $\mathrm{var}(X)$.

(f) Let $U = X + Y$ and $V = X - Y$. Find the joint density of U and V.

(g) Are U and V independent? Justify your answer.

(h) Find the distribution function of X.

(i) Show that $X(2 - X)$ has a $U(0, 1)$ distribution.

9. Exercise 4.17 page 194

10. Exercise 3.28 (c) page 132

11. Exercise 3.29 (c) page 132

12. Exercise 3.32 page 133

(continued)

13. Exercise 2.38 page 82 (modified slightly)
Let X have the negative binomial distribution with pmf

$$f(x) = \binom{r+x-1}{x} p^r (1-p)^x, \quad x = 0, 1, 2, \ldots,$$

where $0 < p < 1$ and $r > 0$ is an integer.

(a) Calculate the mgf of X and use it to find $\mathrm{E}(X)$.

(b) Define a new random variable by $Y = 2pX$. Show that, as $p \downarrow 0$, the mgf of Y converges to that of a chi squared random variable with $2r$ degrees of freedom by showing that

$$\lim_{p \downarrow 0} M_Y(t) = \left(\frac{1}{1-2t} \right)^r, \quad |t| < \frac{1}{2}.$$