Prof. Michael Kozdron

Lecture #1: Introduction to Financial Derivatives

The primary goal of this course is to develop the *Black-Scholes option pricing formula* with a certain amount of mathematical rigour. This will require learning some *stochastic calculus* which is fundamental to the solution of the option pricing problem. The tools of stochastic calculus can then be applied to solve more sophisticated problems in finance and economics. As we will learn, the general Black-Scholes formula for pricing options has had a profound impact on the world of finance. In fact, trillions of dollars worth of options trades are executed each year using this model and its variants. In 1997, Myron S. Scholes (originally from Timmins, ON) and Robert C. Merton were awarded the Nobel Prize in Economics[1] for this work. (Fischer S. Black had died in 1995.)

Exercise 1.1. Read about these Nobel laureates at

`http://nobelprize.org/nobel_prizes/economic-sciences/laureates/1997`

and read the prize lectures *Derivatives in a Dynamic Environment* by Scholes and *Applications of Option-Pricing Theory: Twenty-Five Years Later* by Merton also available from this website.

As noted by McDonald in the Preface of his book *Derivative Markets* [18],

> "Thirty years ago the Black-Scholes formula was new, and derivatives was an esoteric and specialized subject. Today, a basic knowledge of derivatives is necessary to understand modern finance."

Before we proceed any further, we should be clear about what exactly a *derivative* is.

Definition 1.2. A *derivative* is a financial instrument whose value is determined by the value of something else.

That is, a *derivative* is a financial object *derived* from other, usually more basic, financial objects. The basic objects are known as *assets.* According to Higham [11], the term asset is used to describe any financial object whose value is known at present but is liable to change over time. A *stock* is an example of an asset.

A *bond* is used to indicate cash invested in a risk-free savings account earning continuously compounded interest at a known rate.

Note. The term asset does not seem to be used consistently in the literature. There are some sources that consider a derivative to be an asset, while others consider a bond to be an asset. We will follow Higham [11] and use it primarily to refer to stocks (and not to derivatives or bonds).

[1] Technically, Scholes and Merton won *The Sveriges Riksbank Prize in Economic Sciences in Memory of Alfred Nobel.*

Example 1.3. A *mutual fund* can be considered as a derivative since the mutual fund is composed of a range of investments in various stocks and bonds. Mutual funds are often seen as a good investment for people who want to hedge their risk (i.e., diversify their portfolio) and/or do not have the capital or desire to invest heavily in a single stock. Chartered banks, such as TD Canada Trust, sell mutual funds as well as other investments; see

`http://www.tdcanadatrust.com/mutualfunds/mffh.jsp`

for further information.

Other examples of derivatives include *options*, *futures*, and *swaps*. As you probably guessed, our goal is to develop a theory for pricing options.

Example 1.4. An example that is particularly relevant to residents of Saskatchewan is the *Deferred Delivery Contract* of the Canadian Wheat Board (CWB). See

`http://www.cwb.ca/deferred-delivery-contract`

for more information. The basic idea is that a farmer selling, say, barley can enter into a contract in August with the CWB whereby the CWB agrees to pay the farmer a fixed price per tonne of barley in December. The farmer is, in essence, betting that the price of barley in December will be lower that the contract price, in which case the farmer earns more for his barley than the market value. On the other hand, the CWB is betting that the market price per tonne of barley will be higher than the contract price, in which case they can immediately sell the barely that they receive from the farmer for the current market price and hence make a profit. This is an example of an option, and it is a fundamental problem to determine how much this option should be worth. That is, how much should the CWB charge the farmer for the opportunity to enter into an option contract. The Black-Scholes formula will tell us how to price such an option.

Thus, an *option* is a contract entered at time 0 whereby the buyer has the right, but not the obligation, to purchase, at time T, shares of a stock for the fixed value $\$E$. If, at time T, the actual price of the stock is greater than $\$E$, then the buyer exercises the option, buys the stocks for $\$E$ each, and immediately sells them to make a profit. If, at time T, the actual price of the stock is less than $\$E$, then the buyer does not exercise the option and the option becomes worthless. The question, therefore, is "How much should the buyer pay at time 0 for this contract?" Put another way, "What is the *fair price* of this contract?"

Technically, there are *call options* and *put options* depending on one's perspective.

Definition 1.5. A *European call option* gives its *holder* the right (but not the obligation) to purchase from the *writer* a prescribed asset for a prescribed price at a prescribed time in the future.

Definition 1.6. A *European put option* gives its *holder* the right (but not the obligation) to sell to the *writer* a prescribed asset for a prescribed price at a prescribed time in the future.

The prescribed price is known as the *exercise price* or the *strike price*. The prescribed time in the future is known as the *expiry date*.

The adjective *European* is to be contrasted with *American*. While a European option can be exercised only on the expiry date, an *American option* can be exercised at *any* time between the start date and the expiry date. In Chapter 18 of Higham [11], we will see that American call options have the same value as European call options. American put options, however, are more complicated.

Hence, our primary goal will be to systematically develop a fair value of a European call option at time $t = 0$. (The so-called *put-call parity* for European options means that our solution will also apply to European put options.)

Finally, we will use the term *portfolio* to describe a combination of

(i) assets (i.e., stocks),

(ii) options, and

(iii) cash invested in a bank, i.e., bonds.

We assume that it is possible to hold negative amounts of each at no penalty. In other words, we will be allowed to *short sell* stocks and bonds freely and for no cost.

To conclude these introductory remarks, I would like to draw your attention to the recent book *Quant Job Interview Questions and Answers* by M. Joshi, A. Downes, and N. Denson [14]. To quote from the book description,

> "Designed to get you a job in quantitative finance, this book contains over 225 interview questions taken from actual interviews in the City and Wall Street. Each question comes with a full detailed solution, discussion of what the interviewer is seeking and possible follow-up questions. Topics covered include option pricing, probability, mathematics, numerical algorithms and C++, as well as a discussion of the interview process and the non-technical interview."

The "City" refers to "New York City" which is, arguably, the financial capital of the world. (And yes, at least one University of Regina actuarial science graduate has worked in New York City.) Here is an example of the type of question found in this book.

- In the Black-Scholes world, price a European option with a payoff of $\max\{S_T^2 - K, 0\}$ at time T.
- Develop a formula for the price of a derivative paying $\max\{S_T(S_T - K), 0\}$ in the Black-Scholes model.

By the end of the course, you will know how to answer these questions!