Stat 441 Fall 2014
Assignment #2

This assignment is due at the beginning of class on Wednesday, October 1, 2014.

1. Read Chapter 3, pages 21–31, of Higham.

- Everything in this section *should* be familiar from STAT 251 and STAT 252.
- Make sure you can do Exercises 3.1 through 3.9.

2. Read Chapters 5, 6, and 7, pages 45–72, of Higham.

- Chapter 5 is reasonably straightforward.
- Chapters 6 and 7 develop "geometric Brownian motion" as the model of the stock price movement. Read through these chapters to get a feel for what is to come.

3. The following exercises are from the printed lecture notes.

- Exercises 4.5 and 4.7
- Exercise 4.10 is important.

4. Suppose that $\{S_n, n = 0, 1, 2, \ldots\}$ is a simple random walk starting at 0, and consider the filtration $\{\mathcal{F}_n, n = 0, 1, 2, \ldots\}$ given by $\mathcal{F}_n = \sigma(S_0, \ldots, S_n)$. Compute $\mathbb{E}(\sin(S_{n+1})|\mathcal{F}_n)$. Is $\{\sin(S_n), n = 0, 1, 2, \ldots\}$ a martingale. If not, find a function of $\sin(S_n)$ that is a martingale.

5. Suppose that $\{X_j, j = 0, 1, 2, \ldots\}$ is a martingale with respect to the filtration $\{\mathcal{F}_n, n = 0, 1, 2, \ldots\}$. Let $\{Y_j, j = 0, 1, 2, \ldots\}$ be defined as

$$Y_j = f(X_0, \ldots, X_j).$$

Define the process $\{M_n, n = 0, 1, 2, \ldots\}$ by setting $M_0 = 0$ and

$$M_n = \sum_{j=1}^{n} Y_{j-1}(X_j - X_{j-1})$$

for $n = 1, 2, \ldots$.

(a) Verify that $\{M_n, n = 0, 1, 2, \ldots\}$ is a martingale with respect to the filtration $\{\mathcal{F}_n, n = 0, 1, 2, \ldots\}$.

(b) Compute $\mathbb{E}(M_n^2 - M_{n-1}^2|\mathcal{F}_{n-1})$. *Hint: Use* $(M_n - M_{n-1})^2$.

(c) Use the result of (b) to conclude that

$$\mathbb{E}(M_n^2|\mathcal{F}_{n-1}) = \sum_{j=1}^{n} Y_{j-1}^2 \mathbb{E}[(X_j - X_{j-1})^2].$$

(continued)

(d) Use the tower property[1] of conditional expectation to immediately deduce that

$$\mathbb{E}(M_n^2) = \sum_{j=1}^{n} \mathbb{E}(Y_{j-1}^2)\mathbb{E}[(X_j - X_{j-1})^2].$$

6. Complete the MATLAB exercises from Chapters 1, 2, 3, and 5. I'm not sure how straightforward P5.2 is.

7. Write MATLAB programs to simulate a simple random walk of 100 steps, 1000 steps, and 10000 steps.

[1]This is the property that says that the expectation of a conditional expectation is the unconditional expectation, i.e., $\mathbb{E}(\mathbb{E}(Y|X)) = \mathbb{E}(Y)$.