Statistics 351 Midterm #2 – November 20, 2009

This exam has 5 problems on 5 numbered pages and is worth a total of 50 points.

You have 50 minutes to complete this exam. Please read all instructions carefully, and check your answers. Show all work neatly and in order, and clearly indicate your final answers. Answers must be justified whenever possible in order to earn full credit. ***Unless otherwise specified, no credit will be given for unsupported answers, even if your final answer is correct.*** *Points will be deducted for incoherent, incorrect, and/or irrelevant statements.*

This exam is closed-book, except that one $8\frac{1}{2} \times 11$ *double-sided page of handwritten notes is permitted. No other aids are allowed.*

You must answer all of the questions in the space provided. Note that blank space is NOT an indication of a question's difficulty.

Name: ________________________________

Instructor: Michael Kozdron

Problem	Score
1	
2	
3	
4	
5	

TOTAL: ____________

1. (*10 points*) Suppose that X_1, X_2, and X_3 are independent $N(0,1)$ random variables. Set

$$\begin{aligned} Y_1 &= X_1 - X_2 + 1, \\ Y_2 &= 2X_1 + X_2 - 2X_3. \end{aligned}$$

Determine the distribution of $\mathbf{Y} = (Y_1, Y_2)'$.

2. (*10 points*) Let X_1, X_2, X_3, X_4 be independent and identically distributed $U(0,1)$ random variables. As always, let $X_{(j)}$, $j = 1, 2, 3, 4$, denote the jth order variable. Compute

$$P\left(X_{(1)} + X_{(4)} \leq 1\right).$$

3. (*12 points*) Suppose that $X \in U(0,1)$, and that the distribution of Y conditioned on $X = x$ is $N(x, x^2)$; that is, $Y|X = x \in N(x, x^2)$. Compute $E(Y)$, $\mathrm{var}(Y)$, and $\mathrm{cov}(X, Y)$.

Hint: Using properties of conditional expectation will be helpful.

4. (*8 points*) Determine which of the following matrices *cannot* be the covariance matrix of some random vector $\mathbf{X} = (X_1, X_2, X_3)'$:

$$A = \begin{pmatrix} 4 & 2 & 0 \\ 2 & 3 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \quad B = \begin{pmatrix} 4 & 2 & 0 \\ 2 & 3 & -1 \\ 0 & -1 & 1 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}, \quad D = \begin{pmatrix} 3 & 2 & 0 \\ 2 & 3 & 1 \\ 0 & 0 & 1 \end{pmatrix}.$$

Be sure to justify your answers.

5. (*10 points*) Suppose that the random vector $\mathbf{X} = (X_1, X_2)'$ has the multivariate normal distribution

$$\mathbf{X} \in N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix}\right),$$

and that the random vector $\mathbf{Y} = (Y_1, Y_2)'$ has the multivariate normal distribution

$$\mathbf{Y} \in N\left(\begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 & -2 \\ -2 & 3 \end{pmatrix}\right),$$

Suppose further that $\mathbf{X}$ and $\mathbf{Y}$ are independent. Define the random variable Z by setting $Z = (X_1 + X_2) - (Y_1 + Y_2)$. Determine the distribution of Z.