# Statistics 351 Midterm #1 – October 5, 2009

**This exam has 4 problems on 4 numbered pages and is worth a total of 50 points.**

*You have 50 minutes to complete this exam. Please read all instructions carefully, and check your answers. Show all work neatly and in order, and clearly indicate your final answers. Answers must be justified whenever possible in order to earn full credit.* ***Unless otherwise specified, no credit will be given for unsupported answers, even if your final answer is correct.*** *Points will be deducted for incoherent, incorrect, and/or irrelevant statements.*

*This exam is closed-book, except that one* $8\frac{1}{2} \times 11$ *double-sided page of handwritten notes is permitted. No other aids are allowed.*

*You must answer all of the questions in the space provided. Note that blank space is NOT an indication of a question's difficulty.*

Name: ____________________________________

Instructor: Michael Kozdron

| Problem | Score |
|---|---|
| 1 | |
| 2 | |
| 3 | |
| 4 | |

TOTAL: ____________

**1.** (*15 points*) Let $0 < a < b$ be given positive constants, and suppose that the random vector $(X, Y)'$ has joint density function

$$f_{X,Y}(x,y) = \begin{cases} c(y-x), & \text{if } a < x < y < b, \\ 0, & \text{otherwise,} \end{cases}$$

where the value of the normalizing constant $c$ is chosen so that $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{X,Y}(x,y)\,\mathrm{d}x\,\mathrm{d}y = 1$.

**(a)** Determine the value of $c$. (Of course, your answer will depend on $a$ and $b$.)

**(b)** Determine $f_X(x)$, the marginal density function of $X$.

**(c)** Calculate $f_{Y|X=x}(y)$, the conditional density function of $Y$ given $X = x$.

**2.** (*10 points*) The following two transformations define random variables that are of fundamental importance in actuarial applications.

**(a)** Suppose that $X \in N(\mu, \sigma^2)$. Let $Y = e^X$. Determine the density function of $Y$.

**(b)** Suppose that $X \in \Gamma(a, b)$ so that the density of $X$ is $f_X(x) = \dfrac{b^{-a}}{\Gamma(a)} x^{a-1} e^{-x/b}$ for $x > 0$. Let $Y = 1/X$. Determine the density function of $Y$.

**3.** (*15 points*) Suppose that the two-dimensional random vector $(X, Y)'$ has joint density function

$$f_{X,Y}(x, y) = \frac{9}{x^4 y^4}$$

provided that $x > 1$ and $y > 1$. If the random vector $(U, V)'$ is defined by setting

$$U = \sqrt{XY} \quad \text{and} \quad V = \sqrt{\frac{X}{Y}},$$

determine $f_U(u)$, the marginal density function of $U$.

**4.** (*10 points*) Suppose that $(X, Y)'$ is a two-dimensional random vector. It is known that the marginal distribution of $X$ is normal with mean 0 and variance 1; that is $X \in N(0, 1)$. It is also known that the conditional distribution of $Y$ given $X = x$ is normal with mean $x$ and variance 1; that is, $Y|X = x \in N(x, 1)$. Carefully verify that the marginal distribution of $Y$ is normal with mean 0 and variance 2; that is, show $Y \in N(0, 2)$.

*Hint*: You will find it useful to complete the square in the exponent and use the fact that a normal density function integrates to 1.