Stat 351 Fall 2015
Assignment #1

This assignment is due on Wednesday, September 16, 2015. You do not need to submit the problems for grading.

Stat 351 continues the introduction to the mathematical theory of probability that started in Stat 251. The tools that we will use in this class include integration-by-substitution and integration-by-parts from first year calculus, and multiple integration from second year calculus. The beginning of the term is a good time to review this material. We will not be reviewing techniques of integration when we encounter integrals in this course. Here are a couple of direct computations to help you refresh your skills, and to illustrate the type of integrals we will need to consider.

Single-Variable Calculus

1. Compute $\displaystyle\int_0^\infty e^{-2x}\,\mathrm{d}x$.

2. Compute $\displaystyle\int_0^\infty xe^{-2x}\,\mathrm{d}x$.

3. Compute $\displaystyle\int_0^\infty x^2e^{-2x}\,\mathrm{d}x$.

4. Compute $\displaystyle\int_0^\infty x^3e^{-2x}\,\mathrm{d}x$.

5. Compute $\displaystyle\int_0^\infty x^{-2/3}e^{-x^{1/3}}\,\mathrm{d}x$.

6. For any $a > 0$, compute $\displaystyle\int_0^\infty x^{1/a-1}e^{-x^{1/a}}\,\mathrm{d}x$.

7. Compute $\displaystyle\int_0^\infty x^{1/3}e^{-2x^{1/3}}\,\mathrm{d}x$.

8. Compute $\displaystyle\int_0^\infty xe^{-x^2}\,\mathrm{d}x$.

9. For any $a > 0$, compute $\displaystyle\int_0^\infty xe^{-ax^2}\,\mathrm{d}x$.

10. Compute $\displaystyle\int_0^1 x(1-x)^3\,\mathrm{d}x$.

11. Compute $\displaystyle\int_0^1 x^2(1-x)^3\,\mathrm{d}x$.

12. Compute $\displaystyle\int_{-\infty}^\infty \frac{1}{x^2+1}\,\mathrm{d}x$.

13. Compute $\displaystyle\int_0^\infty \frac{x}{x^2+1}\,\mathrm{d}x$.

14. Compute $\displaystyle\int_{-\infty}^{\infty} \frac{x}{x^2+1}\,\mathrm{d}x$.

15. For any $a > 0$, compute $\displaystyle\int_{a}^{\infty} \frac{1}{x^3}\,\mathrm{d}x$.

16. For any $a > 0$ and for any $b > 1$, compute $\displaystyle\int_{a}^{\infty} \frac{1}{x^b}\,\mathrm{d}x$.

Multi-Variable Calculus

For each of the following, compute the value of

$$\iint_R f(x,y)\,\mathrm{d}x\,\mathrm{d}y$$

where R is the region indicated.

1. $f(x,y) = x^2$ and $R = \{0 < x < y < 1\}$.
2. $f(x,y) = x^2$ and $R = \{0 < y < x < 1\}$.
3. $f(x,y) = y^2$ and $R = \{0 < x < y < 1\}$.
4. $f(x,y) = xy$ and $R = \{0 < y < x < 1\}$.
5. $f(x,y) = x + y$ and $R = \{0 < x < y < 1\}$.
6. $f(x,y) = e^{-2y}$ and $R = \{0 < x < 2y\}$.
7. $f(x,y) = \sqrt{x^2+y^2}$ and $R = \{x^2+y^2 \le 1\}$. (Hint: use polar coordinates)
8. $f(x,y) = xy$ and $R = \{x^2+y^2 \le 1,\ x > 0,\ y > 0\}$. (Hint: use polar coordinates)

Some Sums

1. Compute the exact value of $\displaystyle\sum_{j=0}^{\infty} 3^{-j}$. (Hint: It is a particular geometric series.)
2. Compute the exact value of $\displaystyle\sum_{j=1}^{\infty} j3^{-j}$. (You may not have seen this one before.)
3. Compute the exact value of $\displaystyle\sum_{j=0}^{\infty} \frac{3^{-j}}{j!}$. (Hint: It is a particular power series.)