Stat 351 Fall 2015
Assignment #8

Solutions should be completed, but not submitted, by Monday, December 7, 2015.

1. Suppose that $X_1, X_2, \ldots, X_n$ are independent $N(0,1)$ random variables. Define the random vector $\mathbf{X} = (X_1, \ldots, X_n)'$. Determine the distribution of $\mathbf{X}'\mathbf{X}$.

2. Suppose that X and Y are independent $N(0,1)$ random variables.

(a) Compute $P(3X + 4Y > 5)$.

(b) Compute $P(\min\{X, Y\} < 1)$.

(b) Compute $P(|\min\{X, Y\}| < 1)$.

(d) Compute $P(\max\{X, Y\} - \min\{X, Y\} < 1)$.

(e) Compute $P(X^2 + Y^2 \leq 1)$.

Note that you will need to use a table of normal probabilities (or R or SAS) to answer parts (a) through (d).

3. Suppose that the random vector $\mathbf{X} = (X_1, X_2)'$ has the multivariate normal distribution

$$\mathbf{X} \in N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}\right)$$

where $\rho = \mathrm{cov}(X_1, X_2) > 0$.

(a) Prove that there exists a standard normal random variable $Z \in N(0,1)$ such that

$$X_1 = \rho X_2 + \sqrt{1 - \rho^2} Z.$$

(b) Prove that Z is independent of X_2.

4. Exercise 5.3, page 126

5. Chapter 5 Problems, pages 140–145, #2, #4, #11, #12, #13, #15, #16, #25, #39