Stat 351 Fall 2015
Assignment #2

Solutions should be completed, but not submitted, by Friday, September 25, 2015.

1. Let X and Y be independent random variables with $X \in \text{Unif}[1,3]$ and $Y \in \mathcal{N}(0,1)$.

(a) Determine $F_{X,Y}(x,y)$, the joint distribution function of $(X,Y)'$.

(b) Show directly (by computing the indicated partial derivative) that

$$\frac{\partial^2}{\partial x \partial y} F_{X,Y}(x,y) = f_X(x) f_Y(y).$$

Is this surprising? Why or why not?

(c) If $Z \in \text{Exp}(4)$ is independent of X and Y, determine the joint density of $(X,Y,Z)'$.

2. *Here is a STAT 251 example to show that uncorrelated random variables need not be independent.*

Suppose that $X \in \mathcal{N}(0,1)$. Let Y be independent of X with $P\{Y = 1\} = P\{Y = -1\} = 1/2$. Define the random variable Z be setting $Z = XY$.

(a) Compute $\text{cov}(X,Z)$.

(b) Show that $P\{Z \geq 1\} = P\{X \geq 1\}$. Use this fact to conclude that Z and X are NOT independent.

(c) Generalize part (b) to show that $P\{Z \geq x\} = P\{X \geq x\}$ for every $x \in \mathbb{R}$. This implies that $Z \in \mathcal{N}(0,1)$.

3. Fill in the details of Exercise 1.2 and show that X and Y are not independent, even though they are uncorrelated.

4. Do Exercise 1.3 which is similar in spirit to Example 1.1. However, the limits of integration are much easier to handle.

5. Try Exercise 1.1. Finding the marginal distribution of $(X,Y)'$ involves a computation similar to Example 1.1. However, finding the marginal distribution of just X is much more frustrating.

6. Suppose that X_1, X_2, and X_3 are independent and identically distributed continuous random variables with common density function $f(x)$.

(a) Compute $P\{X_1 > X_2\}$.

(b) Compute $P\{X_1 > X_2 | X_1 > X_3\}$.

(c) Compute $P\{X_1 > X_2 | X_1 < X_3\}$.

Hint: You can answer this problem easily using symmetry.

7. Chapter 1 Problems, pages 24–29, #1 through #13, #17 through #43