Statistics 351 Fall 2009 Quiz #1 – Solutions

1. Let C be the event that an automobile owner purchases collision coverage and let D be the event that an automobile owner purchases disability coverage. We know that $P(C) = 2P(D)$ by (i), we know that $P(C \cap D) = P(C) \cdot P(D)$ by (ii), and we know that $P(C \cap D) = 0.15$ by (iii). We then compute

$$0.15 = P(C \cap D) = P(C) \cdot P(D) = 2P(D) \cdot P(D) = 2[P(D)]^2$$

implying that $P(D) = \sqrt{0.15/2}$. Thus, the probability that an automobile owner purchases neither collision nor disability coverage is

$$\begin{aligned} P(C^c \cap D^c) &= P(C^c)P(D^c) = [1 - P(D)][1 - P(C)] = 1 - P(C) - P(D) + P(C)P(D) \\ &= 1 - 2P(D) - P(D) + 2[P(D)]^2 = 1 - 3\sqrt{0.15/2} + 0.15 \doteq 0.3284162. \end{aligned}$$

The answer is (B). (Problem #11)

2. The probability of at least one claim during a particular month, given that there have been at most four claims during that month, is

$$P(N \ge 1 | N \le 4) = \frac{P(N \ge 1,\ \ N \le 4)}{P(N \le 4)} = \frac{P(1 \le N \le 4)}{P(N \le 4)} = \frac{\frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \frac{1}{30}}{\frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \frac{1}{30}} = \frac{2}{5}.$$

The answer is (B). (Problem #24)

3. The expected value of X is given by

$$\begin{aligned} E(X) &= \int_{-2}^{4} x \cdot \frac{|x|}{10} dx = \int_{-2}^{0} x \cdot \frac{|x|}{10} dx + \int_{0}^{4} x \cdot \frac{|x|}{10} dx = \int_{-2}^{0} -\frac{x^2}{10} dx + \int_{0}^{4} \frac{x^2}{10} dx \\ &= -\frac{x^3}{30}\Big|_{-2}^{0} + \frac{x^3}{10}\Big|_{0}^{4} = \frac{-8}{30} + \frac{64}{30} = \frac{28}{15}. \end{aligned}$$

The answer is (D). (Problem #45)

4. If Y denotes the payment for hospitalization, then $E(Y)$ is given by

$$\begin{aligned} E(Y) &= 100P(Y = 100) + 200P(Y = 200) + 300P(Y = 300) + 325P(Y = 325) + 350P(Y = 350) \\ &= 100P(X = 1) + 200P(X = 2) + 300P(X = 3) + 325P(X = 4) + 350P(X = 5) \\ &= 100 \cdot \frac{5}{15} + 200 \cdot \frac{4}{15} + 300 \cdot \frac{3}{15} + 325 \cdot \frac{2}{15} + 350 \cdot \frac{1}{15} \\ &= \frac{3200}{15} \\ &= 213.\overline{3}. \end{aligned}$$

The answer is (D). (Problem #49)

5. We find

$$E(X) = \frac{d}{dt} M_X(t)\Big|_{t=0} = \frac{10000}{(1-2500t)^{-5}}\Big|_{t=0} = 10000$$

and

$$E(X^2) = \frac{d^2}{dt^2} M_X(t)\Big|_{t=0} = \frac{5 \cdot 2500 \cdot 10000}{(1-2500t)^{-6}}\Big|_{t=0} = 5 \cdot 2500 \cdot 10000$$

so that

$$\text{var}(X) = E(X^2) - [E(X)]^2 = 5 \cdot 2500 \cdot 10000 - 10000^2 = 10000 \cdot (5 \cdot 2500 - 10000) = \frac{10000^2}{4}.$$

Thus, the standard deviation of X is $10000/2 = 5000$. The answer is (B). (Problem #57)

6. The 70$^{\text{th}}$ percentiles of X, denoted α, satisfies

$$\int_{200}^{\alpha} \frac{2.5(200)^{2.5}}{x^{3.5}} dx = 0.70$$

while the 30$^{\text{th}}$ percentiles of X, denoted β, satisfies

$$\int_{200}^{\beta} \frac{2.5(200)^{2.5}}{x^{3.5}} dx = 0.30.$$

Thus,

$$0.70 = \int_{200}^{\alpha} \frac{2.5(200)^{2.5}}{x^{3.5}} dx = -\frac{(200)^{2.5}}{x^{2.5}}\Big|_{200}^{\alpha} = 1 - \frac{(200)^{2.5}}{\alpha^{2.5}}$$

so that

$$\alpha = \frac{200}{0.30^{1/2.5}} \doteq 323.7289.$$

Similarly,

$$\beta = \frac{200}{0.70^{1/2.5}} \doteq 230.6698.$$

Thus, the difference between the 30$^{\text{th}}$ and 70$^{\text{th}}$ percentiles of X is $\alpha - \beta \doteq 93.0591$. The answer is (B). (Problem #59)