Statistics 351 Midterm #2 – November 16, 2007

This exam is worth 50 points.

There are 5 problems on 5 numbered pages.

You have 50 minutes to complete this exam. Please read all instructions carefully, and check your answers. Show all work neatly and in order, and clearly indicate your final answers. Answers must be justified whenever possible in order to earn full credit. ***Unless otherwise specified, no credit will be given for unsupported answers, even if your final answer is correct.*** *Points will be deducted for incoherent, incorrect, and/or irrelevant statements.*

This exam is closed-book, except that one $8\frac{1}{2} \times 11$ *double-sided page of handwritten notes is permitted. No other aids are allowed.*

You must answer all of the questions in the space provided. Note that blank space is NOT an indication of a question's difficulty.

Name: ______________________________

Instructor: Michael Kozdron

Problem	Score
1	
2	
3	
4	
5	

TOTAL: __________

1. (*10 points*) Suppose that the random vector $\mathbf{X} = (X_1, X_2)'$ has a multivariate normal distribution with mean vector $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Lambda}$ given by

$$\boldsymbol{\mu} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad \text{and} \quad \boldsymbol{\Lambda} = \begin{pmatrix} 2 & 2 \\ 2 & 3 \end{pmatrix}.$$

(a) Let $Y_1 = X_1 - X_2 - 2$ and $Y_2 = X_1 + X_2$. Determine the distribution of the random vector $\mathbf{Y} = (Y_1, Y_2)'$.

(b) Determine the density function $f_{\mathbf{Y}}(y_1, y_2)$ of $\mathbf{Y}$.

(c) Determine the distribution of $Y_2 | Y_1 = 0$.

2. (*10 points*) Let $\mathbf{X} = (X_1, X_2)'$ be multivariate normal with mean vector $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Lambda}$ where

$$\boldsymbol{\mu} = \begin{pmatrix} 5 \\ 10 \end{pmatrix} \quad \text{and} \quad \boldsymbol{\Lambda} = \begin{pmatrix} 1 & \alpha \\ \alpha & 4 \end{pmatrix}$$

If $Y_1 = 2X_1 + X_2 + 1$ and $Y_2 = 3X_1 - 2X_2 - 2$ are independent, determine the value of α.

3. (*10 points*) Determine which of the following matrices *cannot* be the covariance matrix of some random vector $\mathbf{X} = (X_1, X_2, X_3)'$:

$$A = \begin{pmatrix} 4 & -2 & 0 \\ -2 & 3 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \quad B = \begin{pmatrix} 4 & 2 & 0 \\ 2 & 3 & -1 \\ 0 & -1 & 1 \end{pmatrix}, \quad C = \begin{pmatrix} 2 & 2 & 2 \\ 2 & 2 & 2 \\ 2 & 2 & 2 \end{pmatrix},$$

$$D = \begin{pmatrix} 1 & 2 & 0 \\ 2 & 3 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad E = \begin{pmatrix} 1 & -1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad F = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

Be sure to justify your answers.

4. (*10 points*) Suppose that the random vector $\mathbf{X} = (X_1, X_2)'$ has the multivariate normal distribution

$$\mathbf{X} \in N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix} \right),$$

and that the random vector $\mathbf{Y} = (Y_1, Y_2)'$ has the multivariate normal distribution

$$\mathbf{Y} \in N\left(\begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 & -2 \\ -2 & 3 \end{pmatrix} \right),$$

Suppose further that $\mathbf{X}$ and $\mathbf{Y}$ are independent. It can be shown that if $Z_1 = X_1 + Y_1$ and $Z_2 = X_2 - Y_2$, then the random vector $\mathbf{Z} = (Z_1, Z_2)'$ has a multivariate normal distribution. Determine the mean vector $\boldsymbol{\mu}$ and covariance matrix $\mathbf{\Lambda}$ of $\mathbf{Z}$.

5. (*10 points*) Let X_1, X_2, X_3 be independent Exp(1) random variables. Compute

$$P(X_{(1)} = X_1, X_{(2)} = X_2, X_{(3)} = X_3).$$