Statistics 351 Midterm #1 – October 18, 2006

This exam has 4 problems and 5 numbered pages.

You have 50 minutes to complete this exam. Please read all instructions carefully, and check your answers. Show all work neatly and in order, and clearly indicate your final answers. Answers must be justified whenever possible in order to earn full credit. ***Unless otherwise specified, no credit will be given for unsupported answers, even if your final answer is correct.*** *Points will be deducted for incoherent, incorrect, and/or irrelevant statements.*

This exam is closed-book, except that one $8\frac{1}{2} \times 11$ *double-sided page of handwritten notes is permitted. No other aids are allowed.*

You must answer all of the questions in the space provided. Note that blank space is NOT an indication of a question's difficulty.

Name: ____________________________________

Instructor: Michael Kozdron

Problem	Score
1	
2	
3	
4	

TOTAL: ____________

1. (*24 points*) Suppose that (X, Y) is a point uniformly distributed in that part of the first quadrant which lies within the circle of radius 2. That is, the joint density function of (X, Y) is

$$f_{X,Y}(x, y) = \begin{cases} \frac{1}{\pi}, & \text{if } x > 0,\ y > 0,\ x^2 + y^2 < 4, \\ 0, & \text{otherwise.} \end{cases}$$

(a) Calculate $f_{X|Y=y}(x)$, the conditional density function of X given $Y = y$.

(b) Use the result of **(a)** to calculate $E(X|Y = y)$.

(continued)

Recall that the joint density function of (X, Y) is $f_{X,Y}(x, y) = \begin{cases} \frac{1}{\pi}, & \text{if } x > 0,\ y > 0,\ x^2 + y^2 < 4, \\ 0, & \text{otherwise.} \end{cases}$

Let $U = \sqrt{X^2 + Y^2}$ and $V = \arctan\left(\frac{Y}{X}\right)$.

(c) Determine $f_{U,V}(u, v)$, the joint density function of (U, V).

(d) Determine the marginal density functions $f_U(u)$ and $f_V(v)$, and prove that the random variables U and V are independent.

2. (*8 points*) Suppose that M is a random variable with distribution function

$$F_M(m) = \begin{cases} 0, & \text{if } m \le 0, \\ m^3(4-3m), & \text{if } 0 < m < 1, \\ 1, & \text{if } m \ge 1. \end{cases}$$

Suppose further that $X|M = m \in U(0, m)$ with $M \in F_M$. Determine $f_X(x)$, the density function for X.

3. (*8 points*) Suppose that X_1 and X_2 are independent random variables with $X_1 \in \mathrm{Exp}(1)$ and $X_2 \in \mathrm{Exp}(1/2)$. Let

$$X_{(1)} = \min\{X_1, X_2\}.$$

Compute $P(X_{(1)} = X_1)$.

(*Recall that if* $X \in \mathrm{Exp}(\lambda)$, *then* $f_X(x) = \frac{1}{\lambda} e^{-x/\lambda}$ *for* $x > 0$.)

4. (*10 points*) Suppose that X_1 and X_2 are random variables with $E(X_1) = 1$, $E(X_2) = 2$, and $\text{cov}(X_1, X_2) = 6$. Suppose that X_3 is a random variable with $E(X_3) = 3$ which is independent of both X_1 and X_2. Let $Z = X_1 \cdot X_2 \cdot X_3$ and $Y = X_1 \cdot X_2$.

(a) Compute $E(Z|Y)$.

(b) Compute $E(Z)$.