Stat 351 Fall 2008
Assignment #5

This assignment is due at the beginning of class on Friday, October 3, 2008. You must submit solutions to all problems. As indicated on the course outline, solutions will be graded for both content and clarity of exposition. The solutions that you submit must be neat and orderly. Do not crowd your work or write in multiple columns. Your assignment must be stapled and problem numbers clearly labelled.

1. Suppose that $Y_1, Y_2, \ldots$ are independent and identically distributed random variables with $P\{Y_1 = 1\} = p$, $P\{Y_1 = -1\} = 1 - p$ for some $0 < p < 1/2$. Let $S_n = Y_1 + \cdots + Y_n$ denote their partial sums.

(a) Show that $X_n = S_n - n(2p - 1)$ is a martingale.

(b) Show that $Z_n = X_n^2 - 4np(1 - p) = [S_n - n(2p - 1)]^2 - 4np(1 - p)$ is a martingale.

2. Consider the following sequence $X_0, X_1, X_2, \ldots$ of random variables. Suppose that $p, q \in (0, 1)$ and set $X_0 = p$. Suppose further that the distribution of X_{n+1} depends only on X_n by

$$P(X_{n+1} = (1 - q)X_n \mid X_n) = 1 - X_n,$$

$$P(X_{n+1} = q + (1 - q)X_n \mid X_n) = X_n.$$

Show that $\{X_n,\ n = 0, 1, \ldots\}$ is a martingale.