Stat 351 Fall 2007
Assignment #1 Solutions

2. (a) If $X \sim \text{Unif}[0,2]$, then $F_X(x) = \frac{x}{2}$ for $0 \le x \le 2$, and if $Y \sim \text{Exp}(3)$, then $F_Y(y) = 1 - e^{-y/3}$ for $y > 0$. Since X and Y are independent, we conclude that

$$F_{X,Y}(x,y) = F_X(x) \cdot F_Y(y) = \frac{x}{2}\left(1 - e^{-y/3}\right)$$

for $0 \le x \le 2$ and $y > 0$. We should also note that if $x < 0$, then $F_X(x) = 0$ and if $x \ge 2$, then $F_X(x) = 1$. Furthermore, if $y \le 0$, then $F_Y(y) = 0$. Combining everything we conclude

$$F_{X,Y}(x,y) = \begin{cases} \frac{x}{2}\left(1 - e^{-y/3}\right), & \text{if } 0 \le x \le 2 \text{ and } y > 0, \\ 1 - e^{-y/3}, & \text{if } x > 2 \text{ and } y > 0, \\ 0, & \text{if } x < 0 \text{ or } y \le 0. \end{cases}$$

(b) We find

$$\frac{\partial^2}{\partial x \partial y} F_{X,Y}(x,y) = \frac{\partial^2}{\partial x \partial y}\left[\frac{x}{2}\left(1 - e^{-y/3}\right)\right] = \frac{1}{2} \cdot \frac{1}{3} e^{-y/3}.$$

Since $f_X(x) = \frac{1}{2}$ and $f_Y(y) = \frac{1}{3}e^{-y/3}$, we see that

$$\frac{\partial^2}{\partial x \partial y} F_{X,Y}(x,y) = f_X(x) \cdot f_Y(y)$$

as required.

(c) If $Z \sim \mathcal{N}(0,1)$ is independent of X and Y, then the joint density of (X,Y,Z) is given by

$$f_{X,Y,Z}(x,y,z) = f_X(x) \cdot f_Y(y) \cdot f_Z(z) = \frac{1}{2} \cdot \frac{1}{3} e^{-y/3} \cdot \frac{1}{\sqrt{2\pi}} e^{-z^2/2} = \frac{1}{\sqrt{72\pi}} e^{-\frac{1}{6}(2y+3z^2)}$$

for $0 \le x \le 2$, $y > 0$, and $-\infty < z < \infty$.

3. If X and Y are both discrete random variables, and their joint mass function is $p_{X,Y}(x,y)$, then

$$F_{X,Y}(x,y) = \sum_{x' \le x} \sum_{y' \le y} p_{X,Y}(x',y').$$

If X and Y are both continuous random variables, and their joint density function is $f_{X,Y}(x,y)$, then

$$F_{X,Y}(x,y) = \int_{-\infty}^{x} \int_{-\infty}^{y} f_{X,Y}(u,v)\, dv\, du.$$