Math 312 Midterm Exam – November 8, 2013

This exam is worth 50 points.

This exam has 5 problems and 1 numbered page.

You have 50 minutes to complete this exam. Please read all instructions carefully, and check your answers. Write your solutions in the exam booklets. Show all work neatly and in order, and clearly indicate your final answers. Answers must be justified whenever possible in order to earn full credit. **Unless otherwise specified, no credit will be given for unsupported answers, even if your final answer is correct.** *Points will be deducted for incoherent, incorrect, and/or irrelevant statements.*

This exam is closed-book, except that one $8\frac{1}{2} \times 11$ double-sided page of handwritten notes is permitted. No other aids are allowed.

Instructor: Michael Kozdron

1. (*10 points*) Determine all values $z \in \mathbb{C}$ satisfying $e^{iz} = i$.

2. (*10 points*) Let C denote the contour that is parametrized by $z(t) = 3e^{it}$ for $0 \le t \le \pi/2$. Compute
$$\int_C \frac{1}{|z|^2} \, dz.$$

3. (*10 points*) Suppose that $C = \{|z| = 3\}$ denotes the circle of radius 3 centred at 0 oriented counterclockwise. Compute
$$\int_C \frac{z^2 - 4z + 2}{z - 2} \, dz.$$

4. (*10 points*) Compute
$$\int_C \frac{z}{(z - 1)(z - i)} \, dz$$
where $C = \{|z| = 2\}$ is the circle of radius of radius 2 centred at the origin oriented counterclockwise.

5. (*10 points*) Suppose that $z = x + iy \in \mathbb{C}$ and consider the function
$$f(z) = x^2 + iy^2.$$
Determine where $f(z)$ is differentiable and compute $f'(z)$ at the points of differentiability.