Statistics 296 Midterm – October 9, 2007

This exam has 5 problems and 7 numbered pages and is worth a total of 60 points.

You have 75 minutes to complete this exam. Please read all instructions carefully, and check your answers. Show all work neatly and in order, and clearly indicate your final answers. Answers must be justified whenever possible in order to earn full credit. ***Unless otherwise specified, no credit will be given for unsupported answers, even if your final answer is correct.*** *Points will be deducted for incoherent, incorrect, and/or irrelevant statements.*

This exam is closed-book, except that one $8\frac{1}{2} \times 11$ *double-sided page of handwritten notes is permitted as well as a calculator. No other aids are allowed. A copy of the relevant tables will be provided.*

You must answer all of the questions in the space provided. Note that blank space is NOT an indication of a question's difficulty.

Name: ___________________________________

Instructor: Michael Kozdron

Problem	Score
1	
2	
3	
4	
5	

TOTAL: ____________

1. (*20 points*) A biologist examined the effect of a certain infection on the eating behaviour of mice. Infected apples were offered to a group of 9 mice (experimental group #1), and sterile apples were offered to a group of 5 mice (control group #2). The amounts consumed (measured in grams of apple per kilogram of body weight) are listed in the table below.

Experimental Group	11	33	34	39	44	48	64	112	332
Control Group	80	141	168	215	369				

Let μ_1 denote the true mean of the experimental group and let μ_2 denote the true mean of the control group. Consider testing the hypothesis $H_0 : \mu_1 = \mu_2$ vs. $H_A : \mu_1 \neq \mu_2$ at the significance level $\alpha = 0.15$.

(a) Based on the Mann-Whitney test, is there significance evidence to reject H_0?

(b) Based on the Wilcoxin rank-sum test, is there significance evidence to reject H_0?

(continued)

Suppose that the biologist does not believe that the mean is the best measure of centre to describe these populations. Instead she decides to check whether there is a difference in the population medians of the two groups.

Let θ_1 denote the true median of the experimental group and let θ_2 denote the true median of the control group. Consider testing the hypothesis $H_0 : \theta_1 = \theta_2$ vs. $H_A : \theta_1 \neq \theta_2$ at the significance level $\alpha = 0.15$.

In order to perform the permutation test, she would have to check $\frac{14!}{9!5!} = 2002$ permuted samples. Instead, she decides to collect a random sample of 10 of these permutations. (She includes her actual observations among the 10 permutations.)

(c) Complete the details of the following chart by computing the differences of medians in the last column.

Permuted Sample	Experimental Group	Control Group	Difference of Medians
1	11 33 34 39 44 48 80 168 215	64 112 141 332 369	
2	11 33 39 48 64 80 112 141 215	34 44 168 332 369	
3	11 33 39 44 48 64 168 332 369	34 80 112 141 215	
4	11 34 48 64 80 168 215 332 369	33 39 44 112 141	
5	11 33 48 64 80 112 168 215 332	34 39 44 141 369	
6	33 34 44 80 112 141 215 332 369	11 39 48 64 168	
7	34 39 44 48 80 112 215 332 369	11 33 64 141 168	
8	33 34 48 80 112 141 168 215 332	11 39 44 64 369	
9	11 33 39 44 80 168 215 332 369	34 48 64 112 141	
10*	11 33 34 39 44 48 64 112 332	80 141 168 215 369	

*observed sample

(d) Based on the results of the randomly sampled permutations, is there significance evidence to reject H_0?

2. (*16 points*) The *Bright Idea Lighting Company* manufacturers light bulbs. They claim that over half of their light bulbs last for at least 400 hours. A random sample of 21 light bulbs produced the following lifetimes (in hours):

190 225 265 288 297 303 314 327 335 368 387 392 401 411 426 435 440 441 448 452 463.

Let $\theta_{0.5}$ denote the true median lifetime of *Bright Idea* light bulbs. Consider testing the hypotheses $H_0 : \theta_{0.5} = 400$ vs. $H_A : \theta_{0.5} < 400$ at the $\alpha = 0.10$ significance level.

(a) Based on the (normal approximation to the) binomial test, is there sufficient evidence to reject H_0?

(b) Determine a 95% confidence interval for the median.

(continued)

(c) Determine a 95% confidence interval for the 60th percentile.

(d) Determine a 95% confidence interval for $F(420)$, the probability that a light bulb's lifetime is less than or equal to 420 hours.

3. (*8 points*) An existing flu vaccine is known to be 25% effective in the second year after receiving it. To determine if a new flu vaccine is more effective than the existing flu vaccine, 20 people are chosen at random and vaccinated. If 9 of those receiving the new vaccine do not become sick in the second year, is there significant evidence (at the $\alpha = 0.05$ level) to conclude that the new vaccine is superior to the existing vaccine?

Hint: To answer this question, consider the hypotheses $H_0 : p = 0.25$ vs. $H_A : p > 0.25$ so that the test statistic B, the observed number who did not become sick in the second year, has a binomial distribution with $n = 20$, $p = 0.25$. The p-value for this test is

$$p\text{-value} = P(B \geq 9).$$

Use Table A1 to determine the p-value exactly and make your conclusions accordingly.

4. (*8 points*) Suppose that a certain species of tomato plant has been extensively studied. It has been found that the distribution of the diameters of these tomatoes is roughly bell-shaped with light-tails. In order to test whether a new growth hormone produces larger tomatoes, a botanist decides to conduct a hypothesis test. If μ_1 denotes the true mean of tomatoes with the growth hormone and μ_2 denotes untreated tomatoes, then her hypotheses are $H_0 : \mu_1 = \mu_2$ vs. $H_A : \mu_1 > \mu_2$. She randomly samples 6 tomatoes, applies the new growth hormone to 3 of them, and measures the following diameters (in cm).

Experimental Group	12.7	13.4	13.9
Control Group	9.7	10.4	13.1

(a) Explain why the two-sample t-test is the appropriate hypothesis test for the botanist to perform.

(b) Based on the two sample t-test, is there sufficient evidence to reject H_0 in favour of H_A at the $\alpha = 0.15$ significance level?

5. (*8 points*) Consider the table of standard normal probabilities and let Z be normal with mean 0 and variance 1. We write $\Phi(z)$ to denote the probability $P(Z \leq z)$. In particular, this is formal notation to allow us to determine p-values associated with z-scores.

For instance, a z-score of 1.95 gives $\Phi(1.95) = 0.9750$, a z-score of 1.64 gives $\Phi(1.64) = 0.9495$, and a z-score of 0.90 gives $\Phi(0.90) = 0.8159$.

Suppose that a binomial test is to be conducted for the median from a population which is known to be symmetric. (This implies that the population mean and the population median, while still unknown, are the same, i.e., $\mu = \theta_{0.5}$.)

Consider a binomial test of the median with a null hypothesis of $H_0 : \mu = 75$ and an alternative of $H_A : \mu > 75$. This corresponds to $H_0 : p = 0.5$ where p is the probability that an observation is greater than 75. It can be shown that for a sample of size n, the power of the binomial test (at the $\alpha = 0.05$ significance level) is

$$\text{power} = 1 - \Phi\left(1.645\sqrt{\frac{0.25}{p(1-p)}} - \frac{p - 0.5}{\sqrt{p(1-p)/n}}\right). \qquad (*)$$

In order to determine the power of the binomial test, an alternative for a fixed μ is needed. For example, if $\mu = 75.8$ and $n = 40$, it is known that $p = 0.626$ and so the power is $1 - \Phi(0.053) = 0.48$.

(a) What happens to the power $(*)$ as the alternative μ gets larger, i.e., when p gets smaller?

(b) What happens to the power $(*)$ as the sample size increases (for a fixed choice of p)?