Stat 296 Fall 2007
Partial Solutions to Assignment #1

Stat 151 Review Problems

4. (a) Let X denote the size of an adult male's foot so that X is normally distributed with mean 25 and standard deviation 3. Therefore,

$$P(22 < X < 28) = P\left(\frac{22-25}{3} < \frac{X-25}{3} < \frac{28-25}{3}\right) = P(-1 < Z < 1) \approx 0.6826$$

where $Z \sim \mathcal{N}(0,1)$ and the last equality follows from a normal table.

4. (b) If $\overline{X}$ denotes the average size of an adult male's foot, then X is normally distributed with mean 25 and standard deviation $3/\sqrt{100} = 0.3$. Therefore,

$$P(24.7 < \overline{X} < 25.3) = P\left(\frac{24.7-25}{0.3} < \frac{\overline{X}-25}{0.3} < \frac{25.3-25}{0.3}\right) = P(-1 < Z < 1) \approx 0.6826$$

where $Z \sim \mathcal{N}(0,1)$ and the last equality follows from a normal table as in **(a)** above.

5. (a) Write the values in order: 150, 180, 190, 230, 250, 250, 280, 300, 340, 380. The median is just the mean of the two middle numbers. Since these two numbers are both 250, the median is 250. The mean is a simple calculation: $(150+180+190+230+250+250+280+300+340+380)/10 = 255$. The standard deviation is calculated just as easily:

$$\sqrt{\frac{697300 - \frac{2550^2}{10}}{9}} \approx 72.$$

5. (b) For the Bright Idea Lighting Company, we have

$$P(X > 350) = P\left(Z > \frac{350-262}{41}\right) \approx P(Z > 2.15) \approx 0.0158$$

and for The Electric Company,

$$P(X > 350) \approx P\left(Z > \frac{350-255}{72}\right) \approx P(Z > 1.32) \approx 0.0934$$

where in both cases $Z \sim \mathcal{N}(0,1)$ and using a normal table.

5. (c) An approximate 95% confidence interval for the true mean lifetime of The Electric Company's light bulbs is given by

$$\overline{X} \pm t_{0.025,n-1}\frac{S}{\sqrt{n}} \quad \text{or} \quad 255 \pm 2.262\frac{72}{\sqrt{10}} \quad \text{or} \quad (204, 307).$$

5. (d) Since the mean lifetime of Bright Idea Lighting light bulbs is 262, and since 262 lies in the 95% confidence interval constructed in **(c)**, we conclude that there is *no* significant difference in mean lifetimes for these two companies' light blubs.

6. Let μ_1 denote the mean waiting time for *Cheap-O-Lube* customers last year, and let μ_2 denote the mean waiting time for *Cheap-O-Lube* customers this year. We are interested in testing the hypotheses

$$H_0 : \mu_1 - \mu_2 \leq 0 \quad \text{vs.} \quad H_1 : \mu_1 - \mu_2 > 0.$$

Since there are over 30 data points in each sample, we use a two sample z-test. Thus, our test statistic is

$$Z = \frac{\overline{X}_1 - \overline{X}_2}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}} = \frac{4.5 - 3.5}{\sqrt{\frac{1^2}{200} + \frac{1^2}{180}}} \approx 9.73.$$

From a normal table, the critical value corresponding to $\alpha = 0.05$ is 1.645. Since $9.73 > 1.645$ we reject H_0 and conclude that there is overwhelming evidence to suggest that *Cheap-O-Lube* customers are waiting less this year.