CS 261 Fall 2011
Solutions to Assignment #6

1. You can check your answers with mine. On OCTAVE, one gets

```
> format long
> rand('seed',123456789)
> y=secret('BUBBLEGUM',6)
ans y = 369789376274 369790779302 369795773986 369808264640 369833872346 369880145434
```

You can also check that this is correct.

```
> lagrange([1:6],y,0)
ans = 369789166876
> b10b26(369789166876)
ans = bubblegum
```

2. If $f(x) = x^2$ for $-\pi \le x \le \pi$, then since $x^2 \cos(kx)$ is an even function on $[-\pi, \pi]$, we find

$$a_k = \frac{1}{\pi}\int_{-\pi}^{\pi} x^2 \cos(kx)\,\mathrm{d}x = \frac{2}{\pi}\int_0^{\pi} x^2 \cos(kx)\,\mathrm{d}x$$

for $k = 0, 1, \ldots$. For $k = 0$ we have

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} x^2\,\mathrm{d}x = \frac{2\pi^2}{3}$$

while for $k = 1, 2, \ldots$, we have

$$a_k = \frac{2}{\pi}\int_0^{\pi} x^2 \cos(kx)\,\mathrm{d}x = \frac{4}{k^2}\cos(k\pi) = \frac{4(-1)^k}{k^2}$$

using integration by parts twice. However, since $x^2 \sin(kx)$ is an odd function on $[-\pi, \pi]$, we find

$$b_k = \frac{1}{\pi}\int_{-\pi}^{\pi} x^2 \sin(kx)\,\mathrm{d}x = 0.$$

(a) Therefore, the Fourier series for f is

$$\frac{a_0}{2} + \sum_{k=1}^{\infty}(a_k \cos(kx) + b_k \sin(kx)) = \frac{\pi^2}{3} + \sum_{k=1}^{\infty} \frac{4(-1)^k}{k^2}\cos(kx).$$

(b) Choosing $x = 0$ so that $\cos(kx) = 1$ implies

$$0 = \frac{\pi^2}{3} + \sum_{k=1}^{\infty}\frac{4(-1)^k}{k^2} = \frac{\pi^2}{3} - \sum_{k=1}^{\infty}\frac{4(-1)^{k+1}}{k^2}$$

or, equivalently,

$$\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k^2} = \frac{\pi^2}{12}.$$

Choosing $x = \pi$ so that $\cos(k\pi) = (-1)^k$ implies

$$\pi^2 = \frac{\pi^2}{3} + \sum_{k=1}^{\infty} \frac{4(-1)^k(-1)^k}{k^2} = \frac{\pi^2}{3} + \sum_{k=1}^{\infty} \frac{4}{k^2}$$

so that

$$\sum_{k=1}^{\infty} \frac{1}{k^2} = \frac{1}{4}\left(\pi^2 - \frac{\pi^2}{3}\right) = \frac{\pi^2}{6}.$$