Math 261 Fall 2011
Solutions to Assignment #4

The following four algorithms are used to implement the bisection method, Newton's method, the secant method, and the method of false position, respectively. In order to apply these algorithms, an extra file named `f.m` needs to be created for each problem. Moreover, a second file named `fp.m` is needed for the Newton's method problems.

Bisection Algorithm

```
function p=bisect(a,b,tol,n)
% Output: p estimate of root
% Input: Interval [a,b], Tolerance tol, Max Interations n
% Evaluates a user written function f()

%Check intervals are opposite signs
if f(a)*f(b) >=0
    error('f(a) and f(b) do not have opposite signs')
end

% Initialize Variables
i=1;
fa=f(a) ;

while i < n
   p = a+(b-a)/2;
   fp=f(p);
   if fp==0 | (b-a)/2<tol
       break
   end
   i = i+1;
   if fa*fp > 0
       a=p;
       fa=fp;
   else
       b=p;
   end
   if i== n
       error('Max Iterations Reached, Method Failed')
   end
end
```

Newton's Method Algorithm

```
function p = newton(p0,tol,N)
% Output p: estimate of root
% Input Initial Point p0, Tolerance tol, Max Interations N
% Evaluates user functions f and fp

i=1;
while i < N
    p = p0 - f(p0)/fp(p0);
    if abs(p-p0) < tol
        break
    end
    i = i+1;
    p0=p;
end
i
if i== N
    error('Method Failed.  Maximum Interations Reached')
end
end
```

Secant Method Algorithm

```
function p = secant(p0,p1,tol,N)
%Output Approximate root p
%Input initial values: p0, p1; Tolerance: tol; Max Interations: N
% Evaluates user functions f

i=2;
q0=f(p0);
q1=f(p1);
while i < N
    p = p1-q1*(p1-p0)/(q1-q0);
    if abs(p-p1) < tol
        break
    end
    i=i+1;
    p0=p1;
    q0=q1;
    p1=p;
    q1=f(p);
end
if i== N
    error('Method Failed.  Maximum Interations Reached')
end
end
```

False Position Method Algorithm

```
function p = falsepos(p0,p1,tol,N)
%Output Approximate root p
%Input initial values: p0, p1; Tolerance: tol; Max Interations: N
% Evaluates user functions f

i=2;
q0=f(p0);
q1=f(p1);
while i < N
    p = p1-q1*(p1-p0)/(q1-q0);
    if abs(p-p1) < tol
        break
    end
    i=i+1;
    q = f(p);
    if q*q1 <0
        p0=p1;
        q0=q1;
    end
    p1=p;
    q1=q;
end
if i== N
    error('Method Failed.  Maximum Interations Reached')
end
end
```

1. (Exercise #6 on page 54)

```
% file f.m
function y=f(x)
% y = 3*x - e.^(x);
% y = 2*x + 3*cos(x) - e.^(x);
% y = x.^2 - 4*x + 4 - log(x);
% y = x + 1 - 2*sin(pi*x);
```

(a) If $f(x) = 3x - e^x$, then the bisection method yields that the value of x with $1 \le x \le 2$ for which $f(x) = 0$ is 1.51213836669922.

(b) NOTE: There is an error in the textbook since the function $f(x) = 2x + 3\cos x - e^x$ does not have a root in $[0, 1]$. This function does, however, have a root in $[1, 2]$. Therefore, if $f(x) = 2x + 3\cos x - e^x$, then the bisection method yields that the value of x with $1 \le x \le 2$ for which $f(x) = 0$ is 1.23970794677734.

(c) If $f(x) = x^2 - 4x + 4 - \ln x$, then the bisection method yields that the value of x with $1 \le x \le 2$ for which $f(x) = 0$ is 1.41239166259766, and the value of x with $2 \le x \le 4$ for which $f(x) = 0$ is 3.05710601806641.

(d) If $f(x) = x + 1 - 2\sin(\pi x)$, then the bisection method yields that the value of x with $0 \le x \le 0.5$ for which $f(x) = 0$ is 0.206031799316406, and the value of x with $0.5 \le x \le 1$ for which $f(x) = 0$ is 0.681968688964844.

2. (Exercise #20 on page 55)

```
% file f.m
function y=f(x)
y = -32.17 * ( ( e.^(x) - e.^(-x) )/2 - sin(x) ) - 3.4 * x.^2;
```

Suppose that $x(t)$, the position of the object moving down the smooth inclined plane at time t, is given by

$$x(t) = -\frac{g}{2\omega^2}\left(\frac{e^{\omega t} - e^{-\omega t}}{2} - \sin \omega t\right)$$

where $g = 32.17$ is acceleration due to gravity and $\omega < 0$ is the constant rate of change of angle. (Note that we are told in the problem that $\omega < 0$.) If it is known that $x(1) = 1.7$, then

$$1.7 = -\frac{32.17}{2\omega^2}\left(\frac{e^{\omega} - e^{-\omega}}{2} - \sin \omega\right)$$

so that $\omega < 0$ satisfies

$$0 = -32.17\left(\frac{e^{\omega} - e^{-\omega}}{2} - \sin \omega\right) - 3.4\omega^2.$$

In other words, we need to find the value of $\omega < 0$ that is a root of the function

$$f(\omega) = -32.17\left(\frac{e^{\omega t} - e^{-\omega t}}{2} - \sin \omega t\right) - 3.4\omega^2.$$

Note that while $\omega = 0$ is a root; we want the value of $\omega < 0$ that is a root. Using the bisection method we find the required value of ω is -0.317066978454590.

3. (Exercise #6 on page 75)

```
% file f.m
function y=f(x)
% y = e.^x + 2.^(-x) + 2 * cos(x) - 6;
% y = log(x-1) + cos(x-1);
% y = e.^x - 3*x.^2;
```

```
% file fp.m
function y=f(x)
% y = e.^x + -2*log(2)*2.^(-x) - 2 * sin(x) ;
% y = 1/(x-1) - sin(x-1);
% y = e.^x - 6*x;
```

(a) If $f(x) = e^x + 2^{-x} + 2\cos x - 6$, then Newton's method yields that the value of x with $1 \le x \le 2$ for which $f(x) = 0$ is 1.82938367659478.

(b) If $f(x) = \ln(x-1) + \cos(x-1)$, then Newton's method yields that the value of x with $1.3 \le x \le 2$ for which $f(x) = 0$ is 1.39774630379610.

(e) If $f(x) = e^x - 3x^2$, then Newton's method yields that the value of x with $0 \le x \le 1$ for which $f(x) = 0$ is 0.910007572488714, and the value of x with $3 \le x \le 5$ for which $f(x) = 0$ is 3.73307902865469.

4. (Exercise #26 on page 77)

```
% file f.m
function y=f(x)
y = (1+x).^240 -1 -500*x;
```

```
% file fp.m
function y=f(x)
y = 240*(1+x).^239 -500;
```

Consider the annuity due formula

$$A = \frac{P}{i}\left[(1+i)^n - 1\right].$$

If $A = 750\ 000$, $P = 1500$ per month, and $n = 20 \times 12 = 240$ months, then i satisfies

$$750000 = \frac{1500}{i}\left[(1+i)^{240} - 1\right] \quad \text{or, equivalently,} \quad 500i = (1+i)^{240} - 1.$$

Therefore, consider the function

$$f(i) = (1+i)^{240} - 1 - 500i.$$

Note that $i = 0$ is a root of this equation, although the annuity due formula is not well-defined at $i = 0$. This means that we need to find a positive value of i with $f(i) = 0$. Using the Newton's method algorithm gives $i = 0.00555078190408201$. Thus, the minimal monthly interest rate that the engineer needs is 0.5551%. Note that Newton's method requires an initial position. You should try several to verify that your answer is correct, say $p_0 = 0.1$, $p_0 = 0.5$, and $p_0 = 1$; all three appear to converge to the same answer. An equivalent way to express your answer is as the annual interest rate which is $12 \times i = 0.0666093828489841$, or roughly 6.67%.

5. (Exercise #8 on page 75)

```
% file f.m
function y=f(x)
% y = 2*x *cos(2*x) - (x-2).^2;
% y = (x-2).^2 - log(x);
% y = sin(x) -e.^(-x);
```

(c) If $f(x) = 2x\cos 2x - (x-2)^2$, then the secant method yields that the value of x with $2 \le x \le 3$ for which $f(x) = 0$ is 2.37068691766230, and the value of x with $3 \le x \le 4$ for which $f(x) = 0$ is 3.72211277342042.

(d) If $f(x) = (x-2)^2 - \ln x$, then the secant method yields that the value of x with $1 \le x \le 2$ for which $f(x) = 0$ is 1.41239117202398, and the value of x with $e \le x \le 4$ for which $f(x) = 0$ is 3.05710355003280.

(f) If $f(x) = \sin x - e^{-x}$, then the secant method yields that the value of x with $0 \le x \le 1$ for which $f(x) = 0$ is 0.588532742347889, the value of x with $3 \le x \le 4$ for which $f(x) = 0$ is 3.09636393243154, and the value of x with $6 \le x \le 7$ for which $f(x) = 0$ is 6.28504927338281.

6. (Exercise #12 on page 76)

```
% file f.m
function y=f(x)
% y = x.^2 - 4*x + 4 - log(x);
% y = x + 1 - 2*sin(pi * x);
```

(a) If $f(x) = x^2 - 4x + 4 - \ln x$, then the method of false position yields that the value of x with $1 \le x \le 2$ for which $f(x) = 0$ is 1.41239239819215, and the value of x with $2 \le x \le 4$ for which $f(x) = 0$ is 3.05709936532003. Note that this is the same function as in (d) of the previous problem.

(b) If $f(x) = x + 1 - 2\sin \pi x$, then the method of false position yields that the value of x with $0 \le x \le 1/2$ for which $f(x) = 0$ is 0.206036107124860, and the value of x with $1/2 \le x \le 1$ for which $f(x) = 0$ is 0.681972659174388.