Stat 252 Winter 2016
Assignment #4

This assignment is due at the beginning of class on Monday, February 22, 2016. Your solutions will be graded based on both correctness *and* exposition. In particular, neatness and grammar count. You must write out solutions using full sentences (including capital letters to start sentences and periods to end them) and no abbreviations. That is, symbols such as $\therefore$ and $\Rightarrow$ are forbidden; write out the full words *therefore* and *implies* in their place.

1. Suppose that $Y_1, \dots, Y_n$ is a random sample from a population having common density function

$$f(y|\theta) = \frac{\theta^2}{y^3} \exp\{-\theta/y\}, \quad y > 0,$$

for some parameter $\theta > 0$. Determine $\hat{\theta}_{\text{MLE}}$, the maximum likelihood estimator of θ.

2. Suppose that $Y_1, \dots, Y_n$ is a random sample from a population having common density function $f(y|\theta) = (\theta+1)y^\theta$, $0 \le y \le 1$, for some parameter $\theta > 0$.

(a) Determine $\hat{\theta}_{\text{MOM}}$, the method of moments estimator of θ.

(b) Determine $\hat{\theta}_{\text{MLE}}$, the maximum likelihood estimator of θ.

3. A continuous random variable Y is said to have the Rayleigh(θ) distribution if the probability density function of Y is

$$f(y|\theta) = \frac{y}{\theta^2} \exp\left\{-\frac{y^2}{2\theta^2}\right\}, \quad y > 0,$$

where $\theta > 0$ is a parameter. It can be shown that

$$\mathbb{E}(Y) = \sqrt{\frac{\pi}{2}}\,\theta \quad \text{and} \quad \mathbb{E}(Y^2) = 2\theta^2.$$

(a) Determine the Fisher information $I(\theta)$ for the Rayleigh(θ) distribution.

Suppose that $Y_1, Y_2, \dots, Y_n$ is a random sample from a population having a Rayleigh(θ) distribution.

(b) Compute $\hat{\theta}_{\text{MOM}}$, the method of moments estimator of θ.

(c) Compute the variance of $\hat{\theta}_{\text{MOM}}$.

(d) Determine $\hat{\theta}_{\text{MLE}}$, the maximum likelihood estimator of θ.

4. Suppose that $Y_1, \dots, Y_n$ is a random sample from a Unif$(0, 2\theta+1)$ population so that the population density is

$$f(y|\theta) = \frac{1}{2\theta+1}, \quad 0 < y < 2\theta + 1,$$

for some parameter $\theta > -1/2$. Determine $\hat{\theta}_{\text{MLE}}$, the maximum likelihood estimator of θ.