Stat 252 Winter 2016
Assignment #3

This assignment is due at the beginning of class on Wednesday, February 10, 2016. Your solutions will be graded based on both correctness *and* exposition. In particular, neatness and grammar count. You must write out solutions using full sentences (including capital letters to start sentences and periods to end them) and no abbreviations. That is, symbols such as $\therefore$ and $\Rightarrow$ are forbidden; write out the full words *therefore* and *implies* in their place.

1. Suppose that the random variable Y has density given by $f(y|\theta) = \theta^2 e^{-\theta^2 y}$, $y > 0$, for some parameter $\theta > 0$. Calculate the Fisher information $I(\theta)$.

2. Suppose that $Y_1, \ldots, Y_n$ are a random sample from a population having common density function

$$f(y|\theta) = \frac{\theta^2}{y^3} \exp\{-\theta/y\}, \quad y > 0,$$

for some parameter $\theta > 0$. Compute the Fisher information $I(\theta)$ in a single observation from this population.

3. Suppose that $Y_1, \ldots, Y_n$ are a random sample from a population having common density function

$$f(y|\theta) = \frac{y^2}{2\,\theta^3} \exp\{-y/\theta\}, \quad y > 0,$$

where $\theta > 0$ is a parameter. Let

$$\hat{\theta} := \frac{\overline{Y}}{3}.$$

(a) Show that $\hat{\theta}$ is an unbiased estimator of θ.

(b) Compute the Fisher information $I(\theta)$ in a single observation from this population.

(c) Use the Cramér-Rao inequality to carefully verify that $\hat{\theta}$ is the minimum variance unbiased estimator of θ.

4. Suppose that $Y_1, \ldots, Y_n$ are a random sample from a population having common density function

$$f(y|\theta) = \frac{\theta^{-252}}{251!}\, y^{251}\, e^{-y/\theta}, \quad y > 0,$$

where $\theta > 0$ is a parameter. It is known that if $Y \sim f(y|\theta)$, then $\mathbb{E}(Y) = 252\,\theta$ and $\mathrm{Var}(Y) = 252\,\theta^2$. Let $\hat{\theta} := \overline{Y}$.

(a) Find a function of $\hat{\theta}$ which is an unbiased estimator of θ. Call it $\hat{\theta}_1$

(b) Compute the Fisher information $I(\theta)$ in a single observation from this population.

(c) Use the Cramér-Rao inequality to carefully verify that $\hat{\theta}_1$ is the minimum variance unbiased estimator of θ.