Statistics 252 Midterm #2 – March 16, 2007

This exam has 5 problems and is worth 40 points. **Instructor:** Michael Kozdron

You must answer all of the questions in the exam booklet provided.

You have 50 minutes to complete this exam. Please read all instructions carefully, and check your answers. Show all work neatly and in order, and clearly indicate your final answers. Answers must be justified whenever possible in order to earn full credit. ***Unless otherwise specified, no credit will be given for unsupported answers, even if your final answer is correct.*** *Points will be deducted for incoherent, incorrect, and/or irrelevant statements.*

This exam is closed-book, except that one $8\frac{1}{2} \times 11$ *double-sided page of handwritten notes is permitted. No other aids are allowed.*

You may find the following facts useful.

If $Z \sim \mathcal{N}(0,1)$, then $P(Z > 1.645) \approx 0.05$ and $P(Z > 1.96) \approx 0.025$.

1. (*6 points*) Suppose that the random variable Y has the Uniform$(\theta, 2\theta)$ distribution where $\theta > 0$ is a parameter. Use the pivotal method to verify that if $0 < \alpha < 1$, then

$$\left[\frac{2Y}{4-\alpha}, \frac{2Y}{2+\alpha}\right]$$

is a confidence interval for θ with coverage probability $1 - \alpha$.

2. (*18 points*) Suppose that $Y_1, \ldots, Y_n$ are independent and identically distributed random variables with each Y_i having density function

$$f_Y(y|\theta) = 3\,\theta\,y^2 \exp\left\{-\theta\,y^3\right\}, \quad 0 \le y < \infty,$$

where $\theta > 0$ is a parameter.

- **(a)** Does this density $f_Y(y|\theta)$ belong to an exponential family?
- **(b)** Compute the likelihood function $L(\theta)$ for this random sample.
- **(c)** Show that the maximum likelihood estimator of θ is $\hat{\theta}_{\text{MLE}} = \dfrac{n}{\sum_{i=1}^{n} Y_i^3}$.
- **(d)** Show that $\sum_{i=1}^{n} Y_i^3$ is a sufficient statistic for the estimation of θ.
- **(e)** Explain why $\hat{\theta}_{\text{MLE}}$ must also be a sufficient statistic for the estimation of θ.
- **(f)** Find the Fisher information $I(\theta)$ in a single observation from this density.
- **(g)** Construct an approximate 95% confidence interval for θ based on $\hat{\theta}_{\text{MLE}}$.

3. (*4 points*) Suppose that $Y_1, \ldots, Y_n$ are independent and identically distributed random variables with each Y_i having density function

$$f_Y(y|\theta) = \theta\,y^{\theta-1}, \quad 0 < y < 1,$$

where $\theta > 0$ is a parameter. Determine $\hat{\theta}_{\text{MOM}}$, the method of moments estimator of θ.

4. (*6 points*) Let $Y_1, \ldots, Y_n$ be independent and identically distributed $\mathcal{N}(\mu, \sigma^2)$ random variables where μ is unknown and $\sigma^2 = 16$ is known. You are interested in testing $H_0 : \mu = 0$ against $H_A : \mu > 0$ by rejecting H_0 when $\overline{Y} > 7.84/\sqrt{n}$. Determine the significance level of this hypothesis test.

5. (*6 points*)

- **(a)** Carefully state the Cramer-Rao inequality.
- **(b)** Explain how the Cramer-Rao inequality can be used to determine whether or not an unbiased estimator of θ is the minimum variance unbiased estimator of θ.