Statistics 252 (Winter 2007)
Other Sampling Distributions
January 17, 2007

1. Suppose that the life times of light bulbs are known to be exponentially distributed with parameter $\lambda = 5$ (measured in weeks of continuous illumination). That is, if Y denotes the life time of a randomly selected light bulb, then the density function of Y is

$$f_Y(y) = \frac{1}{5}e^{-y/5} \quad \text{for } y > 0.$$

If $Y_1, \ldots, Y_n$ are a random sample of such light bulbs, determine

(a) the probability that they all last as least 7 weeks, and

(b) the probability that they all die within 16 weeks.

2. Suppose that $Y_1, Y_2, \ldots, Y_{27}$ are a random sample of exponential random variables with parameter $\theta > 0$. That is, the density of each Y_i is

$$f(y) = \frac{1}{\theta}e^{-y/\theta}, \quad y > 0.$$

(a) Show that $\hat{\theta}_1 = Y_{27}$ is an unbiased estimator of θ.

(b) Show that $\hat{\theta}_2 = 27 \cdot \min\{Y_1, \ldots, Y_{27}\}$ is an unbiased estimator of θ.

(c) Show that $\hat{\theta}_3 = \overline{Y}$ is an unbiased estimator of θ.

(d) Which of the three unbiased estimators given in **(a)**, **(b)**, and **(c)** is preferable for the estimation of θ? Justify your answer.