Stat 252 Winter 2007
Hypothesis Tests for Normal Populations

1. Suppose that $Y_1, \ldots, Y_n$ are i.i.d. from the $\mathcal{N}(\mu, \sigma^2)$ distribution where σ^2 is known, but μ is unknown. Consider testing $H_0 : \mu = \mu_0$ against $H_A : \mu > \mu_0$ by rejecting H_0 if $Z > 1.65$ where

$$Z = \frac{\overline{Y} - \mu_0}{\sigma/\sqrt{n}}.$$

Last class we showed that this test has significance level 0.05.

(a) Assume that $\mu_0 = 0$, $\sigma^2 = 25$, and $n = 4$. What is the power of the test when $\mu = 1$, when $\mu = 2$, and when $\mu = 3$?

(b) Repeat part **(a)** assuming a larger sample size of $n = 16$. Do you have any comments about the comparison of power for the two sample sizes?

2. Suppose that $Y_1, \ldots, Y_{10}$ are i.i.d. from the $\mathcal{N}(\mu, \sigma^2)$ distribution where both μ and σ^2 are unknown. As usual, let S^2 denote the sample variance. Consider testing $H_0 : \sigma^2 = 1$ against $H_A : \sigma^2 > 1$ by rejecting H_0 when $S^2 > c$.

(a) Determine c so that this test has significance level 0.1.

(b) What is the power of this test when $\sigma^2 = 2$ and when $\sigma^2 = 3$?

NOTE: Table 6 in the back of the textbook is not complete enough for these calculations, so I have provided some quantiles of the χ^2_9 distribution below. You can give your answers to the accuracy permitted by this information.

α	0.95	0.90	0.85	0.80	0.75	0.70	0.65	0.60	0.55	0.50	0.45	0.40	0.35	0.30
χ^2_α	3.33	4.17	4.82	5.38	5.90	6.39	6.88	7.36	7.84	8.34	8.86	9.41	10.01	10.66

α	0.25	0.20	0.15	0.10	0.05
χ^2_α	11.39	12.24	13.29	14.68	16.92